STUDENT HAND BOOK

2023-24

(2-2)

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Vision of the Institute

To be recognized as a premier institution in offering the value based and futuristic quality technical education to meet the technological need of the society.

Mission of the the Institute

- To impart value quality technical education through innovative teaching and learning methods.
- To continuously produce employable technical graduates with advanced technical skills to meet the current and future technological need of the society.
- To prepare the graduate for high learning with emphasis on academic and industrial research.

Vision of the Department:

To promote excellence in technical education and scientific research in electronics and communication engineering for the benefit of society.

Mission of the Department:

- To impart excellent technical education with state of art facilities inculcating values and lifelong learning attitude.
- To develop core competence in our students imbibing professional ethics and team spirit.
- To encourage research benefiting society through higher learning

PEOs:

PEO 1: Establish themselves as successful professionals in their career and higher education in the field of Electronics & Communication Engineering and allied domains through rigorous quality education.

PEO 2: Develop Professionalism, Ethical values, Excellent Leadership qualities, Communication Skills and teamwork in their Professional front and adapt to current trends by engaging in lifelong learning

PEO 3: Apply the acquired knowledge & skills to develop novel technology and products for solving real life problems those are economically feasible and socially relevant

PEO 4: To prepare the graduates for developing administrative acumen, to adapt diversified and multidisciplinary platforms to compete globally.

Quality Policy

Our quality policy is to continuously strive for over-all development of the department and the students. Our policy is to provide best inputs to the students and to develop them to imbibe the spirit of professionalism, dedication & commitment.

Dress Code

We encourage our students to be formally dressed on and off campus. This nurtures the feeling of equality and belongings among the students fraternity.

All students are required to carry Photo Identity card at all the time while in the campus

POs:

PO1: Engineering Knowledge: Apply knowledge of mathematics, natural science, computing, engineering fundamentals and an engineering specialization as specified in WK1 to WK4 respectively to develop to the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, review research literature and analyze complex engineering problems reaching substantiated conclusions with considerationfor sustainable development. (WK1 to WK4)

PO3: Design/Development of Solutions: Design creative solutions for complex engineering problems and design/develop systems/components/processes to meet identified needs with consideration for the public health and safety, whole-life cost, net zero carbon, culture, society and environment as required. (WK5)

PO4: Conduct Investigations of Complex Problems: Conduct investigations of complex engineering problems using research-based knowledge including design of experiments, modelling, analysis & interpretation of data to provide valid conclusions.(WK8).

PO5: Engineering Tool Usage: Create, select and apply appropriate techniques, resources and modern engineering & IT tools, including prediction and modelling recognizing their limitations to solve complex engineering problems. (WK2 and WK6)

PO6: The Engineer and The World: Analyze and evaluate societal and environmental aspects while solving complex engineering problems for its impact on sustainability with reference to economy, health, safety, legal framework, culture and environment.(WK1, WK5, and WK7).

PO7: Ethics: Apply ethical principles and commit to professional ethics, human values, diversity and inclusion; adhere to national & international laws. (WK9)

PO8: Individual and Collaborative Team work: Function effectively as an individual, andas a member or leader in diverse/multi-disciplinary teams.

PO9: Communication: Communicate effectively and inclusively within the engineering community and society at large, such as being able to comprehend and write effective reports and design documentation, make effective presentations considering cultural, language, and learning differences

PO10: Project Management and Finance: Apply knowledge and understanding of engineering management principles and economic decision-making and apply these to one's own work, as a member and leader in a team, and to manage projects and in multidisciplinary environments.

PO11: Life-Long Learning: Recognize the need for, and have the preparation and ability for

i) independent and life-long learning ii) adaptability to new and emerging technologiesand iii) critical thinking in the broadest context of technological change. (WK8)

PSOs:

- Ability to apply concepts of Electronics & Communication Engineering to associated research areas of electronics, communication, signal processing, VLSI, Embedded systems
- Ability to design, analyze and simulate a variety of Electronics & Communication functional elements using hardware and software tools along with analytic skills

A Bird's Eye view about the Institution

CMR Engineering College, popularly known as CMREC is the brain child of the clairvoyant CH.Narasihma Reddy. CMR Engineering College is one of the best engineering Colleges for aspiring engineering students. It is one of the newly established Colleges by CMR Engineering Educational Society. CMR Engineering College was established in 2010 in 10 Acres and built up area of 4,785.78 Sq.m. with a single - minded aim to provide a perfect platform to students in the field of Engineering, Technology for their academic and overall personality development. The college has a very good academic activity which focuses for the campus placement.

The college is approved by the All India Council for Technical Education, New Delhi and is affiliated to JNT University Hyderabad. The CMREC is offering the three under graduate courses in ECE, CSE and MECH, and post graduate course in ECE and CSE.

Today, CMREC has grown in leaps and bounds and it is no wonder that CMREC has become cynosure of the eyes of many, hankering for the distinguished centre of technological learning.

Discipline, Character and Education are the three tenets for which CMREC stands, is certainly the haven where values blend seamlessly to churn out engineers for future.

- Collaborating with Institutions and Industries.
- Promoting research and development programme for the growth of economy.
- Disseminating technical knowledge in the region by continuing education programmes.
- Aiming at continual improvement of all round development of student

Department Profile

The Department of Electronics and Communication engineering of CMR Engineering College was established in the academic year 2010-11 with an annual intake of 120.The intake was increased to 180 from the academic year 2012-13 and later the intake was increased to 240 from the academic year 2013-14. In addition to this intake, the Department has 20% lateral entry students at II B.Tech level.

M.Tech programme was started with 24 intake in the specialization of Embedded Systems from the year 2013-14 and VLSI System Design from the year 2014-15.

The B.Tech (ECE) program is duly approved by the AICTE and Government of Telangana and affiliated to Jawaharlal Nehru Technological University (JNTUH),Hyderabad. Three batches have graduated so far.

Department have 56 faculty and are members of professional bodies like ISTE,IEEE,IETE. Some of the students are the members of IETE student forum and IEEE student branch of the existing

Department. A technical association (ECMRON) of ECE has been formed by the senior students of the department for the benefits of students to impart additional knowledge in the field of E&C Engineering apart from prescribed curriculum.

The Department has well equipped state of art laboratories to gain good knowledge and technical skills in the field of Electronics, Communication, Microwave, VLSI, Digital Signal Processing & Microprocessors & Microcontrollers. The Department periodically organizes seminars, symposia, workshops and guest lectures for the benefit of both the students and the faculty.

Academic Regulations, Course Structure andDetailed Syllabus under Autonomous Status

BACHELOR OF TECHNOLOGY (B.TECH.)

(CMREC – R-22 Regulations)

(Applicable for the batch admitted from 2022-2023)

PRELIMINARY DEFINITIONS AND NOMENCLATURES

AICTE: Means All India Council for Technical Education, New Delhi.

Autonomous Institute: Means an institute designated as Autonomous by University Grants Commission (UGC), New Delhi in concurrence with affiliating University (Jawaharlal Nehru Technological University, Hyderabad) and State Government of Telangana.

Academic Autonomy: Means freedom to an institute in all aspects of conducting its academic programs, granted by UGC for Promoting Excellence.

Academic Council: The Academic Council is the highest academic body of the institute and is responsible for the maintenance of standards of instruction, education and examination within the institute. Academic Council is an authority as per UGC regulations and it has the right to take decisions on all academic matters including academic research.

Academic Year: It is the period necessary to complete an actual course of study within a year. It comprises two main semesters i.e., (one odd + one even) and supplementary semester.

Branch: Means specialization in a program like B.Tech. Degree program in Electronics and communication Engineering, B.Tech degree program in Computer Science and Engineering, etc.

Board of Studies (BOS): BOS is an authority as defined in UGC regulations, constituted by Head of the Organization for each of the departments separately. They are responsible for curriculum design and updation in respect of all the programs offered by a department.

Backlog Course: A course is considered to be a backlog course, if the student has obtained a failure grade (F) in that course.

Basic Sciences: The courses offered in the areas of Mathematics, Physics, Chemistry etc., are considered to be foundational in nature.

Commission: Means University Grants Commission (UGC), New Delhi.

Choice Based Credit System: The credit based semester system is one which provides flexibility in designing curriculum and assigning credits based on the course content and hours of teaching along with provision of choice for the student in the course selection.

Compulsory course: Course required to be undertaken for the award of the degree as per the program.

Continuous Internal Examination: It is an examination conducted towards sessional assessment.

Core: The courses that are essential constituents of each engineering discipline are categorized as professional core courses for that discipline.

Course: A course is a subject offered by a department for learning in a particular semester.

Course Outcomes: The essential skills that need to be acquired by every student through a course.

Credit: A credit is a unit that gives weight to the value, level or time requirements of an academic course. The number of 'Contact Hours' in a week of a particular course determines its credit value. One credit is equivalent to one lecture/tutorial/lab hour per week.

Credit point: It is the product of grade point and number of credits for a course.

Cumulative Grade Point Average (CGPA): It is a measure of cumulative performance of a student over all the completed semesters. The CGPA is the ratio of total credit points secured by a student in various courses in all semesters and the sum of the total credits of all courses in all the semesters. It is expressed up to two decimal places.

Curriculum: Curriculum incorporates the planned interaction of students with instructional content, materials, resources, and processes for evaluating the attainment of Program Educational Objectives.

Department: An academic entity that conducts relevant curricular and co-curricular activities, involving both teaching and non-teaching staff, and other resources in the process of study for a degree.

Dropping from Semester: Student who does not want to register for any semester can apply in writing in prescribed format before the commencement of that semester.

Elective Course: A course that can be chosen from a set of courses. An elective can be Professional Elective and or Open Elective.

Evaluation: Evaluation is the process of judging the academic performance of the student in her/his courses. It is done through a combination of continuous internal assessment and semester end examinations.

Grade: It is an index of the performance of the students in a said course. Grades are indicated by alphabets.

Grade Point: It is a numerical weight allotted to each letter grade on a 10 - point scale.

Honors: An Honors degree typically refers to a higher level of academic achievement at an undergraduate level.

Institute: Means CMR Engineering, Hyderabad unless indicated otherwise by the context.

Massive Open Online Courses (MOOC): MOOC courses inculcate the habit of self-learning. MOOC courses would be additional choices in all the elective group courses.

Minor: Minor are coherent sequences of courses which may be taken in addition to the courses required for the B.Tech. Degree.

Pre-requisite: A specific course or subject, the knowledge of which is required to complete before student register another course at the next grade level.

Professional Elective: It indicates a course that is discipline centric. An appropriate choice of minimum number of such electives as specified in the program will lead to a degree with specialization.

Program: Means, UG degree program: Bachelor of Technology (B.Tech.) and PG degree program: Master of Technology (M.Tech.).

Program Educational Objectives: The broad career, professional and personal goals that every student will achieve through a strategic and sequential action plan.

Project work: It is a design or research based work to be taken up by a student during his/her final year to achieve a particular aim. It is a credit based course and is to be planned carefully by the student.

Re-Appearing: A student can reappear only in the semester end examination for theory component of a course, subject to the regulations contained herein.

Registration: Process of enrolling into a set of courses in a semester of a program.

Regulations: The regulations, common to all B.Tech. Programs offered by Institute, are designated as – CMREC Regulations – R-22 and are binding on all the stakeholders.

Semester: It is a period of study consisting of 15 to 18 weeks of academic work equivalent to normally 90 working days. Odd semester commences usually in July and even semester in December of every year.

Semester End Examinations: It is an examination conducted for all courses offered in a semester at the end of the semester.

Student Outcomes: The essential skill sets that need to be acquired by every student during her/his program of study. These skill sets are in the areas of employability, entrepreneurial, social and behavioral.

University: Means Jawaharlal Nehru Technological University Hyderabad (JNTUH), Hyderabad, is an affiliating University.

Withdraw from a Course: Withdrawing from a course means that a student can drop from a course within the first two weeks of odd or even semester. However, he / she can choose a substitute course in place of it by exercising the option within 5 working days from the date of withdrawal.

FOREWORD

The autonomy is conferred to **CMR Engineering College (CMREC),** Hyderabad by University Grants Commission (UGC), New Delhi based on its performance as well as future commitment and competency to impart quality education. It is a mark of its ability to function independently in accordance with the set norms of the monitoring bodies including JNT University Hyderabad (JNTUH), Hyderabad and AICTE, New Delhi. It reflects the confidence of the affiliating University in the autonomous institution to uphold and maintain standards it expects to deliver on its own behalf. Thus, an autonomous institution is given the freedom to have its own **examination system** and **monitoring mechanism**, independent of the affiliating University but under its observance.

CMREC is proud to win the credence of all the above bodies monitoring the quality in education and has gladly accepted the responsibility of sustaining, if not improving upon the standards and ethics for which it has been striving for more than a decade in reaching its present standing in the arena of contemporary technical education. As a follow up, statutory bodies such as Academic Council and Board of Studies (BOS) are constituted with the guidance of the Governing Body of the institute and recommendations of the JNTUH to frame the regulations, course structure, and syllabi under autonomous status.

The autonomous regulations, course structure, and syllabi have been prepared after prolonged and detailed interaction with several expertise solicited from academics, industry and research, in accordance with the vision and mission of the institute in order to produce a quality engineering graduate to the society.

All the faculty, parents, and students are requested to go through all the rules and regulations carefully. Any clarifications needed are to be sought at appropriate time and from the principal of the institute, without presumptions, to avoid unwanted subsequent inconveniences and embarrassments. The cooperation of all the stake holders is requested for the successful implementation of the autonomous system in the larger interests of the institute and brighter prospects of engineering graduates.

PRINCIPAL

ACADEMIC REGULATIONS (R22) FOR B.TECH REGULAR STUDENTS WITH EFFECT FROM THE ACADEMIC YEAR 2022-23

1.0 Under-Graduate Degree Programme in Engineering & Technology (UGP in E&T)

Jawaharlal Nehru Technological University Hyderabad (JNTUH) offers a 4-year (8 semesters) **Bachelor of Technology** (B.Tech.) degree programme, under Choice Based Credit System (CBCS) at its non-autonomous constituent and affiliated colleges with effect from the academic year **2022-23.**

Eligibility for Admission

Admission to the undergraduate (UG) programme shall be made either on the basis of the merit rank obtained by the qualified student in entrance test conducted by the Telangana State Government (EAMCET) or the University or on the basis of any other order of merit approved by the University, subject to reservations as prescribed by the government from time to time.

The medium of instructions for the entire undergraduate programme in Engineering & Technology will be **English** only.

B.Tech. Programme Structure

A student after securing admission shall complete the B.Tech. programme in a minimum period of **four** academic years (8 semesters), and a maximum period of **eight** academic years (16 semesters) starting from the date of commencement of first year first semester, failing which student shall forfeit seat in B.Tech course. Each student shall secure 160 credits (with CGPA $\geq$ 5) required for the completion of the undergraduate programme and award of the B.Tech. Degree.

UGC/ AICTE specified definitions/ descriptions are adopted appropriately for various terms and abbreviations used in these academic regulations/ norms, which are listed below.

Semester Scheme

Each undergraduate programme is of 4 academic years (8 semesters) with the academic year divided into two semesters of 22 weeks (□ 90 instructional days) each and in each

semester - „Continuous Internal Evaluation (CIE)‟ and „Semester End Examination (SEE)‟ under Choice Based Credit System (CBCS) and Credit Based Semester System (CBSS) indicated by UGC, and curriculum/course structure suggested by AICTE are followed.

Credit Courses

All subjects/ courses are to be registered by the student in a semester to earn credits which shall be assigned to each subject/ course in an L: T: P: C (lecture periods: tutorial periods: practical periods: credits) structure based on the following general pattern.

- One credit for one hour/ week/ semester for Theory/ Lecture (L) courses or Tutorials.
- One credit for two hours/ week/ semester for Laboratory/ Practical (P) courses.

Courses like Environmental Science, Constitution of India, Intellectual Property Rights, and Gender Sensitization Lab are mandatory courses. These courses will not carry anycredits.

Subject Course Classification

All subjects/ courses offered for the undergraduate programme in E&T (B.Tech. degree programmes) are broadly classified as follows. The University has followed almost all the guidelines issued by AICTE/UGC.

S. No.	Broad Course Classification	Course Group/ Category	Course Description
1	Foundation Courses (FnC)	BS – Basic Sciences	Includes Mathematics, Physics and Chemistry subjects
2		ES - Engineering Sciences	Includes Fundamental Engineering Subjects
3		HS – Humanities and Social Sciences	Includes subjects related to Humanities, Social Sciences and Management
4	Core Courses (CoC)	PC – Professional Core	Includes core subjects related to the parent discipline/ department/ branch of Engineering.
5	Elective Courses (EℓC)	PE – Professional Electives	Includes elective subjects related to the parent discipline/ department/ branch of Engineering.
6		OE – Open Electives	Elective subjects which include inter-disciplinary subjects or subjects in an area outside the parent discipline/ department/ branch of Engineering.
7	Core Courses	Project Work	B.Tech. Project or UG Project or UG Major Project or Project Stage I & II
8		Industry Training/ Internship/ Industry Oriented Mini-	Industry Training/ Internship/ Industry Oriented Mini-Project/ Mini-Project/ Skill Development Courses
9		project/ Mini- Project/ Skill Development Courses	
		Seminar	Seminar/ Colloquium based on core contents related to parent discipline/ department/ branch of Engineering.
10	Minor Courses	-	1 or 2 Credit Courses (subset of HS)
11	Mandatory Courses (MC)	-	Mandatory Courses (non-credit)

Course Registration

A „faculty advisor or counselor" shall be assigned to a group of 20 students, who will advise the students about the undergraduate programme, its course structure and curriculum, choice/option for subjects/ courses, based on their competence, progress, pre-requisites and interest.

The academic section of the college invites „registration forms" from students before the beginning of the semester through „on-line registration", ensuring „date and time stamping". The online registration requests for any „current semester" shall be **completed before the commencement of SEEs (Semester End Examinations) of the 'preceding semester'**.

A student can apply for **on-line** registration, **only after** obtaining the „**written approval**" from faculty advisor/counselor, which should be submitted to the college academic section through the Head of the Department. A copy of it shall be retained with the Head of the Department, Faculty Advisor/ Counselor and the student.

A student may be permitted to register for all the subjects/ courses in a semester as specified in the course structure with maximum additional subject(s)/course(s) limited to 6 Credits (any 2 elective subjects), based on **progress** and SGPA/ CGPA, and completion of the „**pre-requisites'** as indicated for various subjects/ courses, in the department course structure and syllabus contents.

Choice for „**additional subjects/courses**", not more than any 2 elective subjects in any Semester, must be clearly indicated, which needs the specific approval and signature of the Faculty Advisor/Mentor/HOD.

If the student submits ambiguous choices or multiple options or erroneous entries during **online** registration for the subject(s) / course(s) under a given/ specified course group/ category as listed in the course structure, only the first mentioned subject/ course in that category will be taken into consideration.

Subject/ course options exercised through **on-line** registration are final and **cannot** be changed or inter-changed; further, alternate choices also will not be considered. However, if the subject/ course that has already been listed for registration by the Head of the Department in a semester could not be offered due to any inevitable or unexpected reasons, then the student shall be allowed to have alternate choice either for a new subject (subject to offering of such a subject), or for another existing subject (subject to availability of seats). Such alternate arrangements will be made by the Head of the Department, with due notification and time-framed schedule, within **a week** after the commencement of class-work for that semester.

Dropping of subjects/ courses may be permitted, only after obtaining prior approval from the faculty advisor/ counselor „within a period of 15 days" from the beginning of the current semester.

Open Electives: The students have to choose three Open Electives (OE-I, II & III) from the list of Open Electives given by other departments. However, the student can opt for an Open Elective subject offered by his own (parent) department, if the student has not registered and not studied that subject under any category (Professional Core, Professional Electives, Mandatory Courses etc.) offered by parent department in any semester. Open Elective subjects already studied should not repeat/should not match with any category (Professional Core, Professional Electives, Mandatory Courses etc.) of subjects even in the forthcoming semesters.

Professional Electives: The students have to choose six Professional Electives (PE-I to VI) from the list of professional electives given.

Subjects/ courses to be offered

A subject/ course may be offered to the students, **only if** a minimum of 15 students opt for it.

More than **one faculty member** may offer the **same subject** (lab/ practical may be included with the corresponding theory subject in the same semester) in any semester. However, selection of choice for students will be based on - „**first come first serve** basis and CGPA criterion" (i.e. the first focus shall be on early **on-line entry** from the student for registration in that semester, and the second focus, if needed, will be on CGPA of the student).

If more entries for registration of a subject come into picture, then the Head of the Department concerned shall decide, whether or not to offer such a subject/ course for **two (or multiple) sections**.

In case of options coming from students of other departments/ branches/ disciplines (not considering **open electives**), first **priority** shall be given to the student of the „**parent department**".

Attendance requirements:

A student shall be eligible to appear for the semester end examinations, if the student acquires a minimum of 75% of attendance in aggregate of all the subjects/ courses (including attendance in mandatory courses like Environmental Science, Constitution of India, Intellectual Property Rights, and Gender Sensitization Lab) for that semester. **Two periods** of attendance for each theory subject shall be considered, if the student appears for the mid-term examination of that subject. **This attendance should also be Included in the attendance uploaded every fortnight in the University Website.**

Shortage of attendance in aggregate up to 10% (65% and above, and below 75%) in each semester may be condoned by the college academic committee on genuine and valid grounds, based on the student"s representation with supporting evidence.

A stipulated fee shall be payable for condoning of shortage of attendance.

Shortage of attendance below 65% in aggregate shall in **NO** case be condoned.

Students whose shortage of attendance is not condoned in any semester are not eligible to take their end examinations of that semester. They get detained and their registration for that semester shall stand cancelled, including all academic credentials (internal marks etc.) of that semester. **They will not be promoted to the next semester**. They may seek re-registration for all those subjects registered in that semester in which the student is detained, by seeking re-admission into that semester as and when offered; if there are any professional electives and/ or open electives, the same may also be re-registered if offered. However, if those electives are not offered in later semesters, then alternate electives may be chosen from the **same** set of elective subjects offered under that category.

A student fulfilling the attendance requirement in the present semester shall not be eligible for readmission into the same class.

Academic Requirements

The following academic requirements have to be satisfied, in addition to the attendance requirements mentioned in Item No. **6**.

A student shall be deemed to have satisfied the academic requirements and earned the credits allotted to each subject/ course, if student secures not less than 35% (14 marks out of 40 marks) in the Continuous Internal Evaluation (CIE), not less than 35% (21 marks out of 60 marks) in the semester end examinations (SEE), and a minimum of 40% (40 marks out of 100 marks) in the sum total of the CIE (Continuous Internal Evaluation) and SEE (Semester End Examination) taken together; in terms of letter grades, this implies securing **'C'** grade or above in that subject/ course.

A student shall be deemed to have satisfied the academic requirements and earned the credits allotted to Real-time Research Project (or) Field Based Research Project (or) Industry Oriented Mini Project (or) Internship (or) Seminar, if the student secures not less than 40% marks (i.e. 40 out of 100 allotted marks) in each of them. The student is deemed to have failed, if he (i) does not submit a report on Industry Oriented Mini Project/Internship, or (ii) not make a presentation of the same before the evaluation committee as per schedule, or (iii) secures less than 40% marks in Real-time Research Project (or) Field Based Research Project (or) Industry Oriented Mini Project (or) Internship evaluations.

A student may reappear once for each of the above evaluations, when they are scheduled again; if the student fails in such „one reappearance" evaluation also, the student has to reappear for the same in the next subsequent semester, as and when it is scheduled.

Promotion Rules:

S. No.	Promotion	Conditions to be fulfilled
1	First year first semester to first year second semester	Regular course of study of first year first semester.
2	First year second semester to Second year first semester	**(i)** Regular course of study of first year second semester. **(ii)** Must have secured at least 20 credits out of 40 credits i.e., 50% credits up to first year second semester from all the relevant regular and supplementary examinations, whether the student takes thoseexaminations or not.
3.	Second year first semester to Second year second semester	Regular course of study of second year first semester.
4	Second year second semester to Third year first semester	**(i)** Regular course of study of second year second semester. **(ii)** Must have secured at least 48 credits out of 80 credits i.e., 60% credits up to second year second semester from all the relevant regular and supplementary examinations, whether the student takes thoseexaminations or not.
5	Third year first semester to Third year second semester	Regular course of study of third year first semester.
6	Third year second semesterto Fourth year first semester	**(i)** Regular course of study of third year second semester. **(ii)** Must have secured at least 72 credits out of 120 credits i.e., 60% credits up to third year second semester from all the relevant regular and supplementary examinations, whether the student takes thoseexaminations or not.
7	Fourth year first semester to Fourth year second semester	Regular course of study of fourth year first semester.

A student (i) shall register for all courses/subjects covering 160 credits as specified and listed in the course structure, (ii) fulfills all the attendance and academic requirements for 160 credits, (iii) earn all 160 credits by securing SGPA 5.0 (in each semester), and CGPA ≥ 5 (at the end of 8 semesters), (iv) **passes all the mandatory courses,** to successfully complete the undergraduate programme. The performance of the student in these 160 credits shall be considered for the calculation of the final CGPA **(at the end of undergraduate programme),** and shall be indicated in the grade card / marks memo of IV-year II semester.

If a student registers for „**extra subjects'** (in the parent department or other departments/branches of Engg.) other than those listed subjects totaling to 160 credits as specified in the course structure of his department, the performances in those „**extra subjects**‟ (although evaluated and graded using the same procedure as that of the required 160 credits) will not be considered while calculating the SGPA and CGPA. For such „**extra subjects'** registered, percentage of marks and letter grade alone will be indicated in the grade card / marks memo as a performance measure, subject to completion of the attendance and academic requirements as stated in regulations Items 6 and 7.1 – 7.4 above.

A student eligible to appear in the semester end examination for any subject/ course, but absent from it or failed (thereby failing to secure **'C'** grade or above) may reappear for that subject/ course in the supplementary examination as and when conducted. In such cases, internal marks (CIE) assessed earlier for that subject/ course will be carried over, and added to the marks to be obtained in the SEE supplementary examination for evaluating performance in that subject.

A student **detained in a semester due to shortage of attendance may be re-admitted in the same semester in the next academic year for fulfillment of academic requirements**. The academic regulations under which a student has been re-admitted shall be applicable. Further, no grade allotments or SGPA/ CGPA calculations will be done for the entire semester in which the student has been detained.

A student detained **due to lack of credits, shall be promoted to the next academic year only after acquiring the required number of academic credits.** The academic regulations under which the student has been readmitted shall be applicable to him.

Evaluation - Distribution and Weightage of Marks

The performance of a student in every subject/course (including practical‟s and Project Stage – I & II) will be evaluated for 100 marks each, with 40 marks allotted for CIE (Continuous Internal Evaluation) and 60 marks for SEE (Semester End-Examination).

In CIE, for theory subjects, during a semester, there shall be two mid-term examinations. Each Mid-Term examination consists of two parts i) **Part – A** for 10 marks, ii) **Part – B** for 20 marks with a total duration of 2 hours as follows:

1. Mid Term Examination for 30 marks:
   a. Part - A : Objective/quiz paper/Short Answers for 10 marks.(5*2=10Marks)
   b. Part - B : Descriptive paper for 20 marks.

The objective/quiz paper is set with multiple choice, fill-in the blanks and match the following type of questions for a total of 10 marks. The descriptive paper shall contain 6 full questions out of which, the student has to answer 4 questions, each carrying 5 marks. The **average of the two Mid Term Examinations** shall be taken as the final marks for Mid Term Examination (for 30 marks).

The remaining 10 marks of Continuous Internal Evaluation are distributed as:

2. Assignment for 5 marks. (**Average of 2 Assignments** each for 5 marks)
3. Subject Viva-Voce/PPT/Poster Presentation/ Case Study on a topic in the concerned subject for 5 marks.

While the first mid-term examination shall be conducted on 50% of the syllabus, the second mid-term examination shall be conducted on the remaining 50% of the syllabus.

Five (5) marks are allocated for assignments (as specified by the subject teacher concerned). The first assignment should be submitted before the conduct of the first mid-term examination, and the second assignment should be submitted before the conduct of the second mid-term examination. The average of the two assignments shall be taken as the final marks for assignment (for 5 marks).

Subject Viva-Voce/PPT/Poster Presentation/ Case Study on a topic in the subject concerned for 5 marks before II Mid-Term Examination.

- The Student, in each subject, shall have to earn 35% of marks (i.e. 14 marks out of 40 marks) in CIE, 35% of marks (i.e. 21 marks out of 60) in SEE and Overall 40% of marks (i.e. 40 marks out of 100 marks) both CIE and SEE marks put together.

The student is eligible to write Semester End Examination of the concerned subject, if the student scores ≥ 35% (14 marks) of 40 Continuous Internal Examination (CIE) marks.

In case, the student appears for Semester End Examination (SEE) of the concerned subject but not scored minimum 35% of CIE marks (14 marks out of 40 internal marks), his performance in that subject in SEE shall stand cancelled inspite of appearing the SEE.

There is NO Computer Based Test (CBT) for R22 regulations.

The details of the end semester question paper pattern are as follows:

The semester end examinations (SEE), for theory subjects, will be conducted for 60 marks consisting of two parts viz. i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of five questions (numbered from 2 to 6) carrying 10 marks each. Each of these questions is from each unit and may contain sub-questions. For each

question there will be an "either" "or" choice, which means that there will be two questions from each unit and the student should answer either of the two questions.

- The duration of Semester End Examination is 3 hours.

For practical subjects there shall be a Continuous Internal Evaluation (CIE) during thesemester for 40 marks and 60 marks for semester end examination. Out of the 40 marksfor internal evaluation:

1. A write-up on day-to-day experiment in the laboratory (in terms of aim, components/procedure, expected outcome) which shall be evaluated for 10 marks
2. **10 marks for viva-voce (**or) tutorial (or) case study (or) application (or) poster presentation of the course concerned.
3. Internal practical examination conducted by the laboratory teacher concerned shall be evaluated for 10 marks.
4. The remaining 10 marks are for Laboratory Project, which consists of the Design (or) Software / Hardware Model Presentation (or) App Development (or) Prototype Presentation submission which shall be evaluated after completion of laboratory course and before semester end practical examination.

The Semester End Examination shall be conducted with an external examiner and the laboratory teacher. The external examiner shall be appointed from the cluster / other colleges which will be decided by the examination branch of the University.

In the Semester End Examination held for 3 hours, total 60 marks are divided and allocated as shown below:

1. 10 marks for write-up
2. 15 for experiment/program
3. 15 for evaluation of results
4. 10 marks for presentation on another experiment/program in the same laboratory course and
5. 10 marks for viva-voce on concerned laboratory course.

- The Student, in each subject, shall have to earn 35% of marks (i.e. 14 marks out of 40 marks) in CIE, 35% of marks (i.e. 21 marks out of 60) in SEE and Overall 40% of marks (i.e. 40 marks out of 100 marks) both CIE and SEE marks put together.

The student is eligible to write Semester End Examination of the concerned subject, if the student scores ≥ 35% (14 marks) of 40 Continuous Internal Examination (CIE) marks.

In case, the student appears for Semester End Examination (SEE) of the concerned subject but not scored minimum 35% of CIE marks (14 marks out of 40 internal marks), his performance in that subject in SEE shall stand cancelled inspite of appearing the SEE.

There shall be an Industry training (or) Internship (or) Industry oriented Mini-project (or) Skill Development Courses (or) Paper presentation in reputed journal (or) Industry Oriented Mini Project in collaboration with an industry of their specialization. Students shall register for this immediately after II-Year II Semester Examinations and pursue it during summer vacation/semester break & during III Year without effecting regular course work. Internship at reputed organization (or) Skill development courses (or) Paper presentation in reputed journal (or) Industry Oriented Mini Project shall be submitted in a report form and presented before the committee in III-year II semester before end semester examination. It shall be evaluated for 100 external marks. The committee consists of an External Examiner, Head of the Department, Supervisor of the Industry Oriented Mini Project (or) Internship etc, Internal Supervisor and a Senior Faculty Member of the Department. There shall be **NO internal marks** for Industry Training (or) Internship (or) Mini-Project (or) Skill Development Courses (or) Paper Presentation in reputed journal (or) Industry Oriented Mini Project.

The UG project shall be initiated at the end of the IV Year I Semester and the duration of the project work is one semester. The student must present Project Stage – I during IV Year I Semester before II Mid examinations, in consultation with his Supervisor, the title, objective and plan of action of his Project work to the departmental committee for approval before commencement of IV Year II Semester. Only after obtaining the approval of the departmental committee, the student can start his project work.

UG project work shall be carried out in two stages: Project Stage – I for approval of project before Mid-II examinations in IV Year I Semester and Project Stage – II during IV Year II Semester. Student has to submit project work report at the end of IV Year II Semester. The project shall be evaluated for 100 marks before commencement of SEE Theory examinations.

For Project Stage – I, the departmental committee consisting of Head of the Department, project supervisor and a senior faculty member shall approve the project work to begin before II Mid-Term examination of IV Year I Semester. The student is deemed to be not eligible to register for the Project work, if he does not submit a report on Project Stage - I or does not make a presentation of the same before the evaluation committee as per schedule.

A student who has failed may reappear once for the above evaluation, when it is scheduled again; if he fails in such „one reappearance" evaluation also, he has to reappear for the same in the next subsequent semester, as and when it is scheduled.

For Project Stage – II, the external examiner shall evaluate the project work for 60 marks and the internal project committee shall evaluate it for 40 marks. Out of 40 internal marks, the departmental committee consisting of Head of the Department, Project Supervisor and a Senior Faculty Member shall evaluate the project work for 20 marks and Project Supervisor shall evaluate for 20 marks. The topics for Industry Oriented Mini Project/ Internship/SDC etc. and the main Project shall be different from the topic already taken. The student is deemed to have failed, if he (i) does not submit a report on the Project, or (ii) does not make a presentation of the same before the External Examiner as per schedule, or (iii) secures less than 40% marks in the sum total of the CIE and SEE taken together.

For conducting viva-voce of project, University selects an external examiner from the list of experts in the relevant branch submitted by the Principal of the College.

A student who has failed, may reappear once for the above evaluation, when it is scheduled again; if student fails in such „one reappearance" evaluation also, he has to reappear for the same in the next subsequent semester, as and when it is scheduled.

A student shall be given only one time chance to re-register for a maximum of two subjects in a semester:

- If the internal marks secured by a student in the Continuous Internal Evaluation marks for 40 (Sum of average of two mid-term examinations consisting of Objective & descriptive parts, Average of two Assignments & Subject Viva-voce/PPT/ Poster presentation/ Case Study on a topic in the concerned subject) are less than 35% and failed in those subjects.

A student must re-register for the failed subject(s) for 40 marks within four weeks of commencement of the class work in next academic year.

In the event of the student taking this chance, his Continuous Internal Evaluation marks for 40 and Semester End Examination marks for 60 obtained in the previous attempt stand cancelled.

Grading Procedure

Grades will be awarded to indicate the performance of students in each Theory Subject, Laboratory/Practicals/ Industry-Oriented Mini Project/Internship/SDC and Project Stage. Based on the percentage of marks obtained (Continuous Internal Evaluation plus Semester End Examination, both taken together) as specified in item 8 above, a corresponding letter grade shall be given.

As a measure of the performance of a student, a 10-point absolute grading system using the following letter grades (as per UGC/AICTE guidelines) and corresponding percentage of marks shall be followed:

% of Marks Secured in a Subject/Course (Class Intervals)	**Letter Grade (UGC Guidelines)**	**Grade Points**
Greater than or equal to 90%	O (Outstanding)	10
80 and less than 90%	A^+ (Excellent)	9
70 and less than 80%	A (Very Good)	8
60 and less than 70%	B^+ (Good)	7
50 and less than 60%	B (Average)	6
40 and less than 50%	C (Pass)	5
Below 40%	F (FAIL)	0

Absent	**Ab**	**0**

A student who has obtained an „**F’** grade in any subject shall be deemed to have „**failed’** and is required to reappear as a „supplementary student" in the semester end examination, as and when offered. In such cases, internal marks in those subjects will remain the same as those obtained earlier.

To a student who has not appeared for an examination in any subject, „**Ab’** grade will be allocated in that subject, and he is deemed to have „**Failed’**. A student will be required to reappear as a „supplementary student" in the semester end examination, as and when offered next. In this case also, the internal marks in those subjects will remainthe same as those obtained earlier.

A letter grade does not indicate any specific percentage of marks secured by the student, but it indicates only the range of percentage of marks.

A student earns Grade Point (GP) in each subject/ course, on the basis of the letter grade secured in that subject/ course. The corresponding „Credit Points" (CP) are computed by multiplying the grade point with credits for that particular subject/ course.

Credit Points (CP) = Grade Point (GP) x Credits …. For a course

A student passes the subject/ course only when **GP ≥ 5 (‘C’ grade or above)**

The Semester Grade Point Average (SGPA) is calculated by dividing the sum of credit points (□CP) secured from all subjects/ courses registered in a semester, by the total number of credits registered during that semester. SGPA is rounded off to **two** decimal places. SGPA is thus computed as

$$\textbf{SGPA} = \{ \textstyle\sum_{i=1}^{N} C_i\, G_i \} / \{ \textstyle\sum_{i=1}^{N} C_i \} \text{ …. } \textbf{For each semester},$$

where „i" is the subject indicator index (considering all subjects in a semester), „N" is the no. of subjects „**registered’** for the semester (as specifically required and listed under the course structure of the parent department), C_i is the no. of credits allotted to the i^{th} subject, and G_i represents the grade points (GP) corresponding to the letter gradeawarded for that i^{th} subject.

The Cumulative Grade Point Average (CGPA) is a measure of the overall cumulative performance of a student in all semesters considered for registration. The CGPA is the ratio of the total credit points secured by a student in **all** registered courses (of 160) in **all** semesters, and the total number of credits registered in **all** the semesters. CGPA is rounded off to **two** decimal places. CGPA is thus computed from the I year II semester onwards at the end of each semester as per the formula

$$\textbf{CGPA} = \{ \textstyle\sum_{j=1}^{M} C_j\, G_j \} / \{ \textstyle\sum^{M} C_j \} \text{ … } \textbf{for all S semesters registered}$$

j=1

(i.e., up to and inclusive of S semesters, S ≥ 2),

where „**M'** is the **total** no. of subjects (as specifically required and listed under the course structure of the parent department) the student has „**registered'** i.e., from the 1st semester onwards up to and inclusive of the 8th semester, „j" is the subject indicator index (takes into account all subjects from 1 to 8 semesters), Cj is the no. of credits allotted to the jth subject, and Gj represents the grade points (GP) corresponding to the letter grade awarded for that jth subject. After registration and completion of I year I semester, the SGPA of that semester itself may be taken as the CGPA, as there are no cumulative effects.

Illustration of calculation of SGPA:

Course/Subject	**Credits**	**Letter Grade**	**Grade Points**	**Credit Points**
Course 1	4	A	8	4 x 8 = 32
Course 2	4	O	10	4 x 10 = 40
Course 3	4	C	5	4 x 5 = 20
Course 4	3	B	6	3 x 6 = 18
Course 5	3	A+	9	3 x 9 = 27
Course 6	3	C	5	3 x 5 = 15
	21			152

SGPA = 152/21 = 7.24

Illustration of Calculation of CGPA up to 3rd Semester:

Semester	**Course/ Subject Title**	**Credits Allotted**	**Letter Grade Secured**	**Corresponding Grade Point (GP)**	**Credit Points (CP)**
I	Course 1	3	A	8	24
I	Course 2	3	O	10	30
I	Course 3	3	B	6	18
I	Course 4	4	A	8	32
I	Course 5	3	A+	9	27
I	Course 6	4	C	5	20
II	Course 7	4	B	6	24
II	Course 8	4	A	8	32
II	Course 9	3	C	5	15
II	Course 10	3	O	10	30
II	Course 11	3	B+	7	21
II	Course 12	4	B	6	24
II	Course 13	4	A	8	32
II	Course 14	3	O	10	30

III	Course 15	2	A	8	16
III	Course 16	1	C	5	5
III	Course 17	4	O	10	40
III	Course 18	3	B+	7	21
III	Course 19	4	B	6	24
III	Course 20	4	A	8	32
III	Course 21	3	B+	7	21
	Total Credits	**69**		**Total Credit Points**	**518**

CGPA = 518/69 = 7.51

The calculation process of CGPA illustrated above will be followed for each subsequent semester until 8th semester. The CGPA obtained at the end of 8th semester will become the final CGPA secured for entire B.Tech. Programme.

For merit ranking or comparison purposes or any other listing, **only** the „**rounded off'** values of the CGPAs will be used.

SGPA and CGPA of a semester will be mentioned in the semester Memorandum of Grades if all subjects of that semester are passed in first attempt. Otherwise the SGPA and CGPA shall be mentioned only on the Memorandum of Grades in which sitting he passed his last exam in that semester. However, mandatory courses will not be taken into consideration.

Passing Standards

A student shall be declared successful or „passed" in a semester, if he secures a GP ≥ 5 (**'C'** grade or above) in every subject/course in that semester (i.e. when the student gets an SGPA 5.0 at the end of that particular semester); and he shall be declared successful or
„passed" in the entire undergraduate programme, only when gets a CGPA

 5.00 (**'C'** grade or above) for the award of the degree as required.

After the completion of each semester, a grade card or grade sheet shall be issued to all the registered students of that semester, indicating the letter grades and credits earned. It will show the details of the courses registered (course code, title, no. of credits, grade earned, etc.) and credits earned. **There is NO exemption of credits in any case.**

Declaration of results

Computation of SGPA and CGPA are done using the procedure listed in 9.6 to 9.9.

For final percentage of marks equivalent to the computed final CGPA, the following

formula may be used.

% of Marks = (final CGPA – 0.5) x 10

Award of Degree

A student who registers for all the specified subjects/ courses as listed in the course structure and secures the required number of 160 credits (with CGPA 5.0), within 8 academic years from the date of commencement of the first academic year, shall be declared to have „**qualified'** for the award of B.Tech. degree in the branch of Engineering selected at the time of admission.

A student who qualifies for the award of the degree as listed in item 12.1 shall be placedin the following classes.

A student with final CGPA (at the end of the undergraduate programme) > 8.00, and fulfilling the following conditions - shall be placed in „**First Class with Distinction'**. However, he

(i) Should have passed all the subjects/courses in „**First Appearance'** within the first 4 academic years (or 8 sequential semesters) from the date of commencement of first year first semester.

(ii) Should not have been detained or prevented from writing the semester end examinations in any semester due to shortage of attendance or any other reason.

A student not fulfilling any of the above conditions with final CGPA > 8 shall be placed in **'First Class'.**

Students with final CGPA (at the end of the undergraduate programme) 7.0 but < 8.00 shall be placed in **'First Class'.**

Students with final CGPA (at the end of the undergraduate programme) 6.00 but < 7.00, shall be placed in „**Second Class'**.

All other students who qualify for the award of the degree (as per item 12.1), with final CGPA (at the end of the undergraduate programme) 5.00 but < 6, shall be placed in „**pass class**".

A student with final CGPA (at the end of the undergraduate programme) < 5.00 willnot be eligible for the award of the degree.

Students fulfilling the conditions listed under item 12.3 alone will be eligible for awardof „**Gold Medal**".

Award of 2-Year B.Tech. Diploma Certificate

1. A student is awarded 2-Year UG Diploma Certificate in the concerned engineering branch on completion of all the academic requirements and earned all the 80 credits (within 4 years from the date of admission) upto B.Tech. II Year II Semester, if the student want to exit the 4-Year B.Tech. Program and *requests for the 2 -Year* B. Tech. (*UG) Diploma Certificate.*

2. The student **once opted and awarded 2-Year UG Diploma Certificate, the student will be permitted to join** in B. Tech. III Year I Semester and continue for completion of remaining years of study for 4-Year B. Tech. Degree ONLY in the next academic year along with next batch students. *However, if any student wishes to continue the study after opting for exit, he/she should register for the subjects/courses in III Year I Semester before commencement of class work for that semester.*

3. *The students, who exit the 4-Year B. Tech. program after II Year of study and wish to re-join the B.Tech. program, must submit the 2 -Year* B. Tech. (*UG) Diploma Certificate awarded to him, subject to the eligibility for completion of Course/Degree.*

4. A student may be permitted to take one year break after completion of II Year II Semester or B. Tech. III Year II Semester (with university permission through the principal of the college well in advance) and can re-enter the course in **next Academic Year in the same college** and complete the course on fulfilling all the academic credentials within a stipulated duration i.e. double the duration of the course (Ex. within 8 Years for 4-Year program).

Withholding of results

If the student has not paid the fees to the University at any stage, or has dues pending due to any reason whatsoever, or if any case of indiscipline is pending, the result of the student may be withheld, and the student will not be allowed to go into the next higher semester. The award or issue of the degree may also be withheld in such cases.

Transitory Regulations

A. For students detained due to shortage of attendance:

1. A Student who has been detained in I year of R20 Regulations due to lack of attendance, shall be permitted to join I year I Semester of R22 Regulations and he is required to complete the study of B.Tech. Programme within the stipulated period of eight academic years from the date of first admission in I Year.
2. A student who has been detained in any semester of II, III and IV years of R20 regulations for want of attendance, shall be permitted to join the corresponding semester of R22 Regulations and is required to complete the study of B.Tech. within the stipulated period of eight academic years from the date of first admission in I Year. The R22 Academic Regulations under which a student has been readmitted shall be applicable to that student from that semester. See rule (C) for further Transitory Regulations.

B. For students detained due to shortage of credits:

3. A student of R20 Regulations, who has been detained due to lack of credits, shallbe promoted to the next semester of R22 Regulations only after acquiring the required number of credits as per the corresponding regulations of his/her first admission. The total credits required are 160 including both R20 & R22 regulations. The student is required to complete the study of B.Tech. within the stipulated period of eight academic years from the year of first admission. The R22 Academic Regulations are applicable to a student from the year of readmission. See rule (C) for further Transitory Regulations.

C. For readmitted students in R22 Regulations:

4. A student who has failed in any subject under any regulation has to pass those subjects in the same regulations.
5. The maximum credits that a student acquires for the award of degree, shall be the sum of the total number of credits secured in all the regulations of his/her study including R22 Regulations. **There is NO exemption of credits in any case**.
6. If a student is readmitted to R22 Regulations and has any subject with 80% of syllabus common with his/her previous regulations, that particular subject in R22 Regulations will be substituted by another subject to be suggested by the University.

Note: If a student readmitted to R22 Regulations and has not studied any subjects/topics in his/her earlier regulations of study which is prerequisite for further subjects in R22 Regulations, the College Principals concerned shall conduct remedial classes to cover those subjects/topics for the benefit of the students.

Student Transfers

There shall be no branch transfers after the completion of admission process.

There shall be no transfers from one college/stream to another within the constituent colleges and units of Jawaharlal Nehru Technological University Hyderabad.

The students seeking transfer to colleges affiliated to JNTUH from various other Universities/institutions have to pass the failed subjects which are equivalent to the subjects of JNTUH, and also pass the subjects of JNTUH which the students have not studied at the earlier institution. Further, though the students have passed some of the subjects at the earlier institutions, if the same subjects are prescribed in different semesters of JNTUH, the students have to study those subjects in JNTUH in spite of the fact that those subjects are repeated.

The transferred students from other Universities/Institutions to JNTUH affiliated colleges who are on rolls are to be provided one chance to write the CBT (for internal marks) in the **equivalent subject(s)** as per the clearance letter issued by the University.

The autonomous affiliated colleges have to provide one chance to write the internal examinations in the **equivalent subject(s)** to the students transferred from other universities/institutions to JNTUH autonomous affiliated colleges who are on rolls, as per the clearance (equivalence) letter issued by the University.

Scope

The academic regulations should be read as a whole, for the purpose of any interpretation.

In case of any doubt or ambiguity in the interpretation of the above rules, the decision of the Vice-Chancellor is final.

The University may change or amend the academic regulations, course structure or syllabi at any time, and the changes or amendments made shall be applicable to all students with effect from the dates notified by the University authorities.

Where the words “he”, “him”, “his”, occur in the regulations, they include “she”,“her”, “hers”.

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ACADEMIC REGULATIONS FOR B.TECH (LATERAL ENTRY SCHEME) FROM THE AY 2023-24

1. Eligibility for the award of B.Tech Degree (LES)

The LES students after securing admission shall pursue a course of study for not less than three academic years and not more than six academic years.

2. The student shall register for 120 credits and secure 120 credits with CGPA ≥ 5 from II year to IV-year B.Tech. Programme (LES) for the award of B.Tech. Degree.

3. The students, who fail to fulfil the requirement for the award of the degree in six academic years from the year of admission, shall forfeit their seat in B.Tech.

4. The attendance requirements of B. Tech. (Regular) shall be applicable to B.Tech. (LES).

5. Promotion rule

S. No	Promotion	Conditions to be fulfilled
1	Second year first semester to second year second semester	Regular course of study of second year first semester.
2	Second year second semester to third year first semester	**(i)** Regular course of study of second year second semester. **(ii)** Must have secured at least 24 credits out of 40 credits i.e., 60% credits up to second year second semester from all the relevant regular and supplementary examinations, whether the student takes those examinations or not.
3	Third year first semester to third year second semester	Regular course of study of third year first semester.
4	Third year second semester to fourth year first semester	**(i)** Regular course of study of third year second semester. **(ii)** Must have secured at least 48 credits out of 80 credits i.e., 60% credits up to third year second semester from all the relevant regular and supplementary examinations, whether the student takes those examinations or not.
5	Fourth year first semester to fourth year second semester	Regular course of study of fourth yearfirst semester.

6. All the other regulations as applicable to B. Tech. 4-year degree course (Regular) will hold good for B. Tech. (Lateral Entry Scheme).

7. LES students are not eligible for 2-Year B. Tech. Diploma Certificate.

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Malpractices Rules

Disciplinary Action For / Improper Conduct in Examinations

	Nature of Malpractices/Improper conduct	**Punishment**
	If the student:	
1. (a)	Possesses or keeps accessible in examination hall, any paper, note book, programmable calculators, cell phones, pager, palm computers or any other form of material concerned with or related to the subject of the examination (theory or practical) in which student is appearing but has not made use of (material shall include any marks on the body of the student which can be used as an aid in the subject of the examination)	Expulsion from the examination hall and cancellation of the performance in that subject only.
(b)	Gives assistance or guidance or receives it from any other student orally or by any other body language methods or communicates through cell phones with any student or persons in or outside the exam hall in respect of any matter.	Expulsion from the examination hall and cancellation of the performance in that subject only of all the students involved. In case of an outsider, he will be handed over to the police and a case is registered against him.
2.	Has copied in the examination hall from any paper, book, programmable calculators, palm computers or any other form of material relevant to the subject of the examination (theory or practical) in which the student is appearing.	Expulsion from the examination hall and cancellation of the performance in that subject and all other subjects the student has already appeared including practical examinations and project work and shall not be permitted to appear for the remaining examinations of the subjects of that semester/year. The hall ticket of the student is to be cancelled and sent to the University.
3.	Impersonates any other student in connection with the examination.	The student who has impersonated shall be expelled from examination hall. The student is also debarred and forfeits the seat. The performance of the original student who has been impersonated, shall be cancelled in all the subjects of the examination (including practicals and project work) already appeared and shall not be allowed to appear for examinations of the remaining subjects of that semester/year. The student is also debarred for two consecutive

		semesters from class work and all University examinations. The continuation of the course by the student is subject to the academic regulations in connection with forfeiture of seat. If the imposter is an outsider, he will be handed over to the police and a case is registered against him.
4.	Smuggles in the answer book or additional sheet or takes out or arranges to send out the question paper during the examination or answer book or additional sheet, during or after the examination.	Expulsion from the examination hall and cancellation of performance in that subject and all the other subjects the student has already appeared including practical examinations and project work and shall not be permitted for the remaining examinations of the subjects of that semester/year. The student is also debarred for two consecutive semesters from class work and all University examinations. The continuation of the course by the student is subject to the academic regulations in connection with forfeiture of seat.
5.	Uses objectionable, abusive or offensive language in the answer paper or in letters to the examiners or writes to the examiner requesting him to award pass marks.	Cancellation of the performance in that subject.
6.	Refuses to obey the orders of the chief superintendent/assistant – superintendent / any officer on duty or misbehaves or creates disturbance of any kind in and around the examination hall or organizes a walk out or instigates others to walk out, or threatens the officer-in charge or any person on duty in or outside the examination hall of any injury to his person or to any of his relations whether by words, either spoken or written or by signs or by visible representation, assaults the officer-in-charge, or any person on duty in or outside the examination hall or any of his relations, or indulges in any other act of misconduct or mischief which result in damage to or destruction of property in the examination hall or any	In case of students of the college, they shall be expelled from examination halls and cancellation of their performance in that subject and all other subjects the student(s) has (have) already appeared and shall not be permitted to appear for the remaining examinations of the subjects of that semester/year. The students also are debarred and forfeit their seats. In case of outsiders, they will be handed over to the police and a police case is registered against them.

	part of the college campus or engages in any other act which in the opinion of the officer on duty amounts to use of unfair means or misconduct or has the tendency to disrupt the orderly conduct of the examination.	
7.	Leaves the exam hall taking away answer script or intentionally tears off the script or any part thereof inside or outside the examination hall.	Expulsion from the examination hall and cancellation of performance in that subject and all the other subjects the student has already appeared including practical examinations and project work and shall not be permitted for the remaining examinations of the subjects of that semester/year. The student is also debarred for two consecutive semesters from class work and all University examinations. The continuation of the course by the student is subject to the academic regulations in connection with forfeiture of seat.
8.	Possesses any lethal weapon or firearm in the examination hall.	Expulsion from the examination hall and cancellation of the performance in that subject and all other subjects the student has already appeared including practical examinations and project work and shall not be permitted for the remaining examinations of the subjects of that semester/year. The student is also debarred and forfeits the seat.
9.	If student of the college, who is not a student for the particular examination or any person not connected with the college indulges in any malpractice or improper conduct mentioned in clause 6 to 8.	Expulsion from the examination hall and cancellation of the performance in that subject and all other subjects the student has already appeared including practical examinations and project work and shall not be permitted for the remaining examinations of the subjects of that semester/year. The student is also debarred and forfeits the seat. Person(s) who do not belong to the college will be handed over to the police and, a police case will be registered against them.
10.	Comes in a drunken condition to the examination hall.	Expulsion from the examination hall and cancellation of the performance in that subject and all other subjects the student has already appeared for including practical examinations and project work and shall not be permitted for

		the remaining examinations of the subjects of that semester/year.
11.	Copying detected on the basis of internal evidence, such as, during valuation or during special scrutiny.	Cancellation of the performance in that subject and all other subjects the student has appeared for including practical examinations and project work of that semester/year examinations.
12.	If any malpractice is detected which is not covered in the above clauses 1 to 11 shall be reported to the University for further action to award a suitable punishment.	

Malpractices identified by squad or special invigilators

1. Punishments to the students as per the above guidelines.
2. Punishment for Institutions: (if the squad reports that the college is also involved inencouraging malpractices)
   a. A show-cause notice shall be issued to the college.
   b. Impose a suitable fine on the college.
   c. Shifting the examination center from one college to another college for a specificperiod of not less than one year.

ACADEMIC CALENDER(2023-24)

CMR ENGINEERING COLLEGE
UGC AUTONOMOUS
(Approved by AICTE - New Delhi. Affiliated to JNTUH and Accredited by NAAC & NBA)
Kandlakoya (V), Medchal (M), Medchal - Malkajgiri (D)-501401

ACADEMIC CALENDAR (REVISED)
B.Tech II-YEAR: ACADEMIC YEAR - 2023-2024

II B.Tech. I – SEMISTER

S. No.	EVENT	DATE FROM	DATE TO	DURATION
1	Commencement of Class Work	18.09.2023		---
2	First Spell of Instructions (Including Dusara Holidays)*	18.09.2023	18.11.2023	9 weeks
3	**First Mid Term Examinations (Theory & Practical)**	**20.11.2023**	**25.11.2023**	***1 Week***
4	Submission of First Mid Term Marks to Exam Branch	02.12.2023		---
5	Parents Teacher's Meeting	09.12.2023		---
6	Second Spell of Instructions (Including Pongal Holidays)	27.11.2023	20.01.2024	8 weeks
7	**Second Mid Term Examinations (Theory & Practical)**	**22.01.2024**	**27.01.2024**	***1 Week***
8	Submission of Second Mid Term Marks to Exam Branch	03.02.2024		---
9	Preparation Holidays and Practical Examinations	29.01.2024	03.02.2024	1 week
10	**End Semester & Supplementary Examinations**	**05.02.2024**	**17.02.2024**	***2 Weeks***

II B.Tech. II – Semester

S. No.	EVENT	DATE FROM	DATE TO	DURATION
1	Commencement of II-SEM Class work	**19.02.2024**		---
2	First Spell of Instructions	19.02.2024	13.04.2024	8 weeks
3	**First Mid Term Examinations**	**15.04.2024**	**20.04.2024**	1 week
4	Submission of First Mid Term Marks to Exam Branch	27.04.2024		---
5	Parents Teacher's Meeting	04.05.2024		
6	Second Spell of Instructions	22.04.2024	11.05.2024	3 weeks
7	**Summer Vacation**	***13.05.2024***	***25.05.2024***	***2 weeks***
8	Continuation of second spell of Instructions	27.05.2024	29.06.2024	5 weeks
9	**Second Mid Term Examinations**	**01.07.2024**	**06.07.2024**	**1 week**
10	Submission of Second Mid Term Marks to Exam Branch	13.07.2024		---
11	Preparation Holidays and Practical Examinations	08.07.2024	13.07.2024	1 week
12	**End Semester & Supplementary Examinations**	**15.07.2024**	**27.07.2024**	***2 weeks***
13	Commencement of Class Work for the next A.Y-2024-2025	*29.07.2024*		---

- *** Dusara Vacation (**Subjected to declaration by JNTUH / TS Govt.)

Controller of Examination
CMR Engineering College
(Autonomous)
Kandlakoya (V), Medchal Dist.,
Hyderabad, T.S. - 501 401.
Controller of Examinations

Principal
Principal
CMR Engineering College
(Autonomous)
Kandlakoya (V), Medchal Dist.

Department Event Planner A.Y 2023-2024

S.NO	DATE	NAMEOFTHEEVENT
1	04/12/2023	Commencement of Class Work for IV Year
2	29/01/2024	Commencement of Class Work for III Year
3	19/02/2024	Commencement of Class Work for II Year
4	04/12/2023-27/01/2024	I Spell of instructions for IV Year
5	29/01/2024-23/03/2024	I Spell of instructions for III Year
6	19/02/2024-13/04/2024	I Spell of instructions for II Year
7	30/12/2024-31/12/24	IV B.Tech Major Project Work Review
9	14/12/2023	Student Workshop-I for III Year
10	15/03/24	Industrial visit
11	10/01/2024-11/01/2024	IV B.Tech Major Project Work Review II
12	29/01/2024-31/01/2024	I MID Exams for IV Year
15	07/08/2024	Guest lecture for III year
16	10/02/2024	Submission of I mid marks for IV Years to University
18	26/03/2024-30/03/2024	IMID Exams for III Year
19	26/03/2024-30/03/2024	IMID Lab Internal Exam for III Year
21	15/04/2024-20/04/2024	IMID Exams for II Year
22	15/03/2024-16/03/2024	IMID Lab Internal Exam for II Year
23	06/04/2024	Submission of I mid marks for III Years to University
24	27/04/2024	Submission of I mid marks for II Years to University
25	06/09/2024	Professional Body Activities
26	01/02/2024-27/03/2024	II Spell of instructions for IV Years (Including I mid examinations)
27	01/04/2024-11/05/2024	II Spell of instructions for III Years (Including I mid examinations)
28	22/04/2024-11/05/2024	II Spell of instructions for II Years(Including I mid examinations)
29	11/03/2024-12/03/24	IV B.Tech Major Project Work Review III
30	28/03/2024-	II MID Exams for IV Years

	30/03/2024	
31	06/04/2024	Marks Submission of II mid for IV Years to University
32	01/04/2024-06/04/2024	Preparation and project evaluation
33	08/04/2024-20/04/2024	End Semester Exams for IV Years
34	12/03/2024	Workshop for II year
35	10/06/2024-15/06/2024	II MID Exams for III Years
36	01/07/2024-06/07/2024	II MID Exams for II Years
37	22/06/2024	Marks Submission of II mid for III Years to University
38	13/07/2024	Marks Submission of II mid for II Years to University
39	01/07/2024-06/07/2024	II Lab Internal Exam for II Year
40	10/06/2024-13/06/2024	II Lab Internal Exam for III Year
41	06/07/2024-10/07/2024	Lab External Exam for II Year
42	15/06/2024-18/06/2024	Lab External Exam for III Year
43	17/06/2024-22/06/2024	Preparation Holidays and Practical Examinations for III years
44	08/07/2024-13/07/2024	Preparation Holidays and Practical Examinations for II years
45	24/06/2024-06/07/2024	End Semester Exams for III Years
46	15/07/2024-27/07/2024	End Semester Exams for II Years

LIST OF SUBJECTS:

1	Electromagnetic fields and Transmission lines
2	Analog and Digital Communication
3	Linear and Digital IC Applications
4	Electronic Circuit Analysis
5	Numerical Techniques and Complex Variables
6	Analog and Digital Communications Laboratory
7	Linear and Digital IC Applications Laboratory
8	Electronic Circuit Analysis Laboratory
9	Real Time Project/Field Based Project
10	Gender Sensitization Lab

ACADEMIC PLANNER

SUBJECT: ELECTRONIC CIRCUIT ANALYSIS

(1) *PREAMBLE/ INTRODUCTION*

This subject provides an insight into analysis and design of different amplifiers like single stage and multistage amplifiers, feedback and non-feedback amplifiers, small signal and large signal amplifiers, oscillators and tuned amplifiers. Amplifiers are the basic blocks of communication system used for design of RF, IF and AF amplifiers.

(2) *PREREQUISITES*

The prerequisites for understanding this course include knowledge of basic semiconductor physics, concepts of BJT, FET transistors and exact h-parameter analysis of different configurations of BJT, FET amplifiers.

Course Code.CO No.	**Course Outcomes (CO's)**	**Blooms Level's**
C225.1	**Extract** the equivalent models for BJT & JFET at low & high frequencies so as to analyze any electronic circuit.	**BL2**
C225.2	**Differentiate** between the positive & negative feedback concepts as applied to various electronic circuits.	**BL4**
C225.3	**Design** and analyze oscillator circuits to generate audio & radio frequency sinusoidal signals.	**BL6**
C225.4	**Realize** different types of power amplifiers for practical applications as per the specifications.	**BL4**
C225.5	**Analyze** various non-sinusoidal signals using different multivibrators for various electronic applications. Apply time basegenerator circuits which is used in applications like CRO & TV.	**BL4**

(3) *OBJECTIVES AND OUTCOMES*

Electronic Circuit Analysis is intended for undergraduate Electronics and communication Engineering students. The purpose of this course is to provide a foundation for analyzing and designing analog electronic circuits.

This course is designed to give an introductory idea of the fundamental aspects of single stage and multistage amplifiers. This learning is enhanced by going through the hand analysis and calculations. Design is the heart of engineering. Good design evolves out of considerable experience with analysis. In this course various characteristics and properties of circuits are planned out as we go through the analysis. The objective is to develop an intuition that can be applied to the design process. Meeting an explicit set of design criteria is the first step in the design process.

This course lays the foundation of Electronics and Communication Engineering. Every topic here is therefore included as part of the syllabus for most of the competitive exams like the IES, GATE etc.

Upon completing this course, the student will be able to

1. Design Multistage amplifiers and understand the concepts of High Frequency Analysis of amplifiers.
2. Understand the concepts of Negative feedback to improve stability of amplifiers.
3. Design Oscillators using Positive feedback amplifiers.
4. Design and realize different Classes of Power Amplifiers and Tuned Amplifiers for audio and Radio applications.
5. Design Multivibrators and Sweep Circuits for various applications.

6. Course Outcome (CO)-Program Outcome (PO) Matrix:

Course Outcomes	PO 1	P02	PO3	PO4	PO 5	PO6	PO 7	PO8	PO 9	PO10	PO1 1	PO1 2
CO1	2	2	-	2	-	-	-	-	-	-	-	3
CO2	2	2	-	2	-	-	-	-	-	-	2	3
CO3	2	3	3	2	-	-	-	-	-	-	2	3
CO4	2	3	3	-	-	-	-	-	-	-	2	3
CO5	2	3	3	-	-	-	-	-	-	-	-	-
AVG	2	3	3	2	-	-	-	-	-	-	2	3

Course Outcome (CO)-Program Specific Outcome (PSO) Matrix:

CO's/ PSO's	PSO1	PSO2
C212.1	2	-
C212.2	2	-
C212.3	2	3
C212.4	2	3
C212.5	2	-
AVG	2	3

7. Justification for Correlation of CO-PO

CO1: Extract the equivalent models for BJT & JFET at low & high frequencies so as to analyze any electronic circuit.
Correlated with PO1 moderately: Because it deals with types of Amplifiers which makes students to acquire engineering knowledge.
Correlated with PO2 moderately: Because it provides students to identify different amplifier models to contributes a solution to complex problems and review research.
Correlated with PO4 moderately: Because students can able to understand the engineering solutions by analyzing Amplifier circuits.
Correlated with PO12 substantially: Because students have to adapt to changes in the Equivalent circuits.

CO2: Differentiate between the positive & negative feedback concepts as applied to various electronic circuits.
Correlated with PO1 moderately: Because it deals with Various feedback amplifiers which make students to acquire engineering knowledge.
Correlated with PO2 moderately: Because it makes students to understand basic feed back concepts which contributes a solution to complex problems and review research.
Correlated with PO4 moderately: : Because students can able to understand the engineering solutions by analyzing.
Correlated with PO11 moderately: Because they can able to understand the feedback concepts.
Correlated with PO12 substantially: Because students have to adapt to changes in Feedback amplifiers.

CO3: Design and analyze oscillator circuits to generate audio & radio frequency sinusoidal signals.
Correlated with PO1 moderately: Because it deals with fundamentals of Oscillators students to acquire engineering knowledge.
Correlated with PO2 substantially: Because it makes students to Analyze different oscillator circuits which contributes a solution to complex problems and review research.
Correlated with PO3 substantially: Because it makes students to design circuit at different frequencies which contribute to design of oscillators.
Correlated with PO4 moderately: Because students can able to understand the engineering solutions by examine the different signals.

Correlated with PO11 moderately: Because they can able to understand the Oscillators.
Correlated with PO12 substantially: Because students have to adapt to changes in Oscillator Design.

CO4: **Realize** different types of power amplifiers for practical applications as per the specifications.
Correlated with PO1 moderately: Because it deals with fundamentals of different types of Amplifiers which makes students to acquire engineering knowledge.
Correlated with PO2 substantially: Because it makes students to analyze characteristics of Power Amplifiers which contributes a solution to complex problems and review research.
Correlated with PO3 substantially: Because it makes students to perform parametric analysis of Amplifiers which contributes to design of Power Amplifiers.
Correlated with PO11 moderately: Because they can able to understand the engineering Principles.
Correlated with PO12 substantially: Because students have to adapt to changes in Amplifier design.

CO5: Analyze various non-sinusoidal signals using different multivibrators for various electronic applications. Apply time basegenerator circuits which is used in applications like CRO & TV.
Correlated with PO1 moderately: Because it deals with multivibrators which makes students to acquire engineering knowledge.
Correlated with PO2 substantially: Because it makes students to identify building blocks of Multivibrators which contributes a solution to complex problems and review research.
Correlated with PO3 substantially: Because it makes students to design Different multivibrator circuits.

8. Justification for Correlation of CO-PSO

CO1: Extract the equivalent models for BJT & JFET at low & high frequencies so as to analyze any electronic circuit.
Correlated with PSO1 moderately: Because it makes students to apply fundamental amplifier circuits.

CO2: Differentiate between the positive & negative feedback concepts as applied to various

electronic circuits.
Correlated with PSO1 moderately: Because it deals with fundamentals of feedback amplifiers which makes students to study Amplifier circuits.

CO3: Design and analyze oscillator circuits to generate audio & radio frequency sinusoidal signals.
Correlated with PSO1 moderately: Because it deals with fundamentals of oscillators which make students to analyze at different frequencies.
Correlated with PSO2 substantially: Because it makes students to design oscillator characteristics Which contribute to find its parameters.

CO4: Realize different types of power amplifiers for practical applications as per the specifications.
Correlated with PSO1 moderately: Because it deals with power amplifiers which make students to interpret characteristics of amplifier circuits.
Correlated with PSO2 substantially: Because it makes students to perform analysis of amplifier which contributes to design of different Amplifiers.

CO5: Analyze various non-sinusoidal signals using different multivibrators for various electronic applications. Apply time basegenerator circuits which is used in applications like CRO & TV.
Correlated with PSO1 moderately: Because it deals with fundamentals of Multivibrators which makes students to build feedback amplifiers.

(4.1) *SYLLABUS - JNTU*

UNIT – I

Multistage Amplifiers: Classification of Amplifiers, Distortion in amplifiers, Different coupling schemes used in amplifiers, Frequency response and Analysis of multistage amplifiers, Cascaded RC Coupled amplifiers, Cascode amplifier, Darlington pair.
Transistor at High Frequency: Hybrid - model of Common Emitter transistor model, fα, fβ and unity gain bandwidth, Gain-bandwidth product.

UNIT-II

Feedback Amplifiers: Concepts of feedback – Classification of feedback amplifiers – General characteristics of Negative feedback amplifiers – Effect of Feedback on Amplifier characteristics –Voltage series, Voltage shunt, Current series and Current shunt Feedback configurations – Simple problems.

UNIT -III

Oscillators: Condition for Oscillations, RC type Oscillators-RC phase shift and Wien-bridge Oscillators, LC type Oscillators –Generalized analysis of LC Oscillators, Hartley and Colpitts Oscillators, Frequency and amplitude stability of Oscillators, Crystal Oscillator.

UNIT -IV

Large Signal Amplifiers: Class A Power Amplifier- Series fed and Transformer coupled, Conversion Efficiency, Class B Power Amplifier- Push Pull and Complimentary Symmetry configurations, Conversion Efficiency, Principle of operation of Class AB and Class –C Amplifiers.

Tuned Amplifiers: Introduction, single Tuned Amplifiers – Q-factor, frequency response of tuned amplifiers, Concept of stagger tuning and synchronous tuning.

UNIT –V

Multivibrators: Analysis and Design of Bistable, Monostable, Astable Multivibrators and Schmitt trigger using Transistors.

Time Base Generators: General features of a Time base Signal, Methods of Generating Time Base Waveform, concepts of Transistor Miller and Bootstrap Time Base Generator, Methods of Linearity improvement.

(4.2) *SYLLABUS - GATE*

UNIT I

Small signal Equivalent circuits of BJTs, Single stage amplifiers. Multi stage amplifiers.

UNIT II

Feedback amplifiers and problems.

UNIT III

Sinusoidal oscillators, criterion for oscillation.

UNIT IV

Power amplifiers.

UNIT V

Multivibrators

(4.3) *SYLLABUS - IES*

UNIT I

Small signal analysis.

UNIT II

Feedback amplifiers.

UNIT III

Oscillators

UNIT IV
Power and Tuned amplifiers.

UNIT V
Multivibrators

(5) *LIST OF EXPERT DETAILS*

The Expert Details which have been mentioned below are only a few of the eminent ones known Internationally, Nationally and Locally. There are a few others known as well.

INTERNATIONAL

1. Prof. Donald Neamen
   Professor and Associate Chairman for the Department of Electrical and Computer Engineering
   The University of New Mexico.
   e-mail: neamen@eece.unm.edu

2. Prof. Anantha P. Chandrakasan
   Professor of Electrical Engineering and Computer Science
   Massachusetts Institute of Technology.
   e-mail: anantha@mtl.mit.edu

NATIONAL

1. Prof. Anindya Sundar Dhar
   Professor Department of Electronics & Electrical Communication Engineering
   IIT Kharagpur.
   e-mail: asd@ece.iitkgp.ernet.in

2. Prof. Indrajit Chakrabarti
   Professor Department of Electronics & Electrical Communication Engineering
   IIT Kharagpur.
   e-mail: indrajit@ece.iitkgp.ernet.in

REGIONAL

1. Dr. Krishna Prasad K S R
   Professor Department of Electronics & Communication Engineering
   NIT Warangal.
   e-mail: krish@nitw.ac.in
2. Dr. K. Lal Kishore
   Dean Research
   CVR Engineering College, Hyderabad.
   e-mail: lalkishorek@yahoo.com
   Mobile: 9618023478

(6) *JOURNALS*

INTERNATIONAL

1. The Journal of Non-Linear Analysis and Application
2. International Journal of Electronics (Taylor & Francis Group)
3. International Journal of Modeling and Simulation (ACTA Press)
4. IEEE Transactions on Circuits and systems
5. IEEE Transactions on Electronic Devices.

NATIONAL

1. Journal of the Institute of Engineers
2. Journal of the Indian Institute of Science
3. IETE Journal of Education
4. IETE Journal of Research
5. IETE Technical Review

LIST OF REFERENCE PAPERS FOR LITERATURE STUDY

1. Dan-Abia, D., Obot, A. And Udofia, K. 2019. Design And Analysis Of A Multistage Common Emitter Amplifier For Low Frequency Applications. *European Journal Of Engineering And Technology Research*. 4, 10 (Oct. 2019), 87-92. Doi:Https://Doi.Org/10.24018/Ejers.2019.4.10.1431.

2. N. K. Kaphungkui “Two Stage Cascade Bjt Amplifier For Very Small Signal Amplification”. *International Research Journal Of Engineering And Technology*. Pp223- 224, Vol 41, 2012.

3. Ayobamidele, Segun & Oyebola, Blessed. (2018). Feedback Amplifier, Its Operation, Effect Importance And Connecting Types: A Review. 16-32.

4. Jyh- Neng Yang, Ming- Jeui Wu, Zen- Chi Hu, Terng- Ren Hsu, Chen – Yi Lee, “Constant – Power Cmos Lc Oscillators Using High Q Active Inductors”, 4th Wseas International Conference On Electronics, Control And Signal Processing, Miami, Florida, Usa, 17-19 Nov. 2005(Pp 105-111)

5. Ali Hajimiri And Thomas H. Lee, “Design Issues In Cmos Differential Lc Oscillator”, Ieee Journal Of Solid-State Circuits, Vol, 34, No.5,1999.

(7) *SUBJECT-LESSON PLAN*

Subject code	Name of the subject	Year/Branch
EC405PC	ELECTRONIC CIRCUIT ANALYSIS	II B.TECH II SEM ECE

S.NO	Topic (JNTU Syllabus)	Sub-Topic	No. of Lectures	Suggested	Remarks

			Required	Books	
UNIT – I Multistage Amplifiers Transistor at High Frequency	Classification of Amplifiers	Different categories of amplifiers	L1	T2, R2	
	Distortion in Amplifiers	Amplitude, Frequency & Phase distortions	L2	T2, R2	
	Different coupling schemes used in amplifiers	RC, Transformer, Direct coupled amplifiers	L3, L4, L5	T1, T2,R1	
	Frequency response and Analysis of multistage amplifiers	Low, Mid, High frequency considerations	L6	T1, T2,R1	
	Cascaded RC Coupled amplifiers,	Design of single Stage RC Coupled Amplifier using BJT	L7	T1, T2,R1	
	Cascode amplifier, Darlington pair	Analysis of Cascode Amplifier	L8	T1, T2,R1	
	The Hybrid pi (Π)	Transconductance model	L9	T1, T2,R4	
	CE Short Circuit Current Gain	Derivation for CE Short Circuit Current Gain	L10	T1, T2,R4	
	Current Gain with Resistive Load	Derivation for Current Gain with Resistive Load	L11	T1, T2,R4	
	Gain Bandwidth Product	Gain Bandwidth Product	L12	T1, T2,R4	
	Emitter Follower at high frequencies	Analysis of Emitter Follower at high frequencies	L13	T1, T2,R4	
	Basic Concepts	Non-linear system analysis	L14	T3, T2	
	TOTAL NO OF CLASSES 14				
UNIT – II	Concepts of	Block diagram of feedback amplifier,	L15,16	T2, R1, R4	

Feedback Amplifiers	Feedback,	Positive & Negative feedbacks			
	Classification of Feedback Amplifiers	Voltage & Current series, Voltage & Current shunt feedback amplifiers	L17	T2, R1, R4	
	General characteristics of Negative Feedback Amplifiers, Effect of Feedback on Amplifier Characteristics	Gain , Bandwidth, Noise, Distortion, Input & output resistances	L18	T2, R1, R4	
	Voltage Series & Current Series Feedback Configuration	Effect of feedback on input and output resistances	L19	T2, R1, R4	
	Voltage Shunt & Current Shunt Feedback Configuration	Effect of feedback on input and output resistances	L20	T2, R1, R4	
	Illustrative Problems	Problems	L21,22	T2, R1, R4	
	TOTAL NO OF CLASSES 8				
UNIT – III Oscillators	Classification of Oscillators, Conditions for Oscillations	Different oscillators, Barkhausen criterion	L23	T1, T2, R1, R4	
	Wein Bridge Oscillators	BJT,FET Wein Bridge oscillators operation	L24,25	T1, T2, R1, R4	
	Generalized analysis of LC Oscillators	Operation of tank circuit	L26,27	T1, T2, R1, R4	
	TOTAL NO OF CLASSES 5				

UNIT – IV Large Signal Amplifiers &Tuned Amplifiers	Classification of Large Signal Amplifiers	Class A, B, C & AB large signal amplifiers	L28,29	T1, T2,R2	
	Class A Large Signal Amplifiers	Efficiency of Series Fed Class A Large Signal Amplifiers	L30	T1, T2,R2	
	Transformer Coupled Class A Audio Power Amplifier	Efficiency of Transformer Coupled Class A Audio Power Amplifier	L31,32	T1, T2,R2	
	Distortion in Power Amplifiers	Three point method second harmonic distortion	L33	T1, T2,R2	
	Class B Push-Pull Amplifier	Analysis of Class B Push-Pull Amplifier	L34	T1, T2,R2	
	Complementary Symmetry Class B Push-Pull Amplifier	Analysis of Complementary Symmetry Class B Push-Pull Amplifier	L35	T1, T2,R2	
	Thermal Stability and Heat Sinks	Condition for Thermal Stability and Heat Sinks	L36	T1, T2,R2	
	Introduction, Q-Factor	Coil Losses, Loaded & Un-Loaded Q	L37	T2, R4	
	Small Signal Tuned Amplifiers	Single & Double tuned amplifiers	L38,39	T2, R4	
	Effect of Cascoding Single Tuned Amplifiers on Bandwidth	Bandwidth equation for Single Tuned Amplifiers	L40	T2, R4	
	Effect of Cascoding Double Tuned Amplifiers on Bandwidth	Bandwidth equation for Double Tuned Amplifiers	L41	T2, R4	
	Stagger Tuned Amplifiers	Analysis of Stagger Tuned Amplifiers	L42	T2, R4	

	Stability of Tuned Amplifiers	Neutralization methods for Stability of Tuned Amplifiers	L43	T2, R4	
TOTAL NO OF CLASSES 16					
Unit V Multivibrators & Time Base Generators	Analysis of Bi-stable Multi vibrators.	Analysis of Bi-stable Multi vibrators.	L44	T1, T2,R2	
	Design of Bi-stable Multi vibrators using Transistors	Design of Bi-stable Multi vibrators using Transistors	L45	T1, T2,R2	
	Analysis of Mono-stable Multi vibrators	Analysis of Mono-stable Multi vibrators	L46	T1, T2,R2	
	Design of Mono-stable Multi vibrators	Design of Mono-stable Multi vibrators	L47	T1, T2,R2	
	Problems on astable,monostable and bistable multivibrators	Problems on astable,monostable and bistable multivibrators	L48	T2, R4	
	Analysis of As table Multi vibrators	Analysis of As table Multi vibrators	L49	T2, R4	
	Design of As table Multi vibrators using Transistors	Design of As table Multi vibrators using Transistors	L50	T2, R4	
	Analysis of Schmitt trigger	Analysis of Schmitt trigger	L51	T1, T2,R2	
	Design of Schmitt trigger using Transistors	Design of Schmitt trigger using Transistors	L52	T1, T2,R2	
	General features of a Time base signal,	General features of a Time base signal,	L53	T1, T2,R2	
	Methods of Generating Time Base Waveform	Methods of Generating Time Base Waveform	L54	T1, T2,R2	
	Miller Time Base Generators-Basic Principle	Miller Time Base Generators-Basic Principle	L55	T1, T2,R2	

	Transistor Miller Time Base generator	Transistor Miller Time Base generator	L56	T1, T2,R2	
	Transistor Bootstrap Time base Generator ,Bootstrap Time Base Generators-Basic Principle	Transistor Bootstrap Time base Generator ,Bootstrap Time Base Generators-Basic Principle	L57	T2, R4	
	Transistor Current Time Base Generators Problems, Methods of Linearity improvement	Transistor Current Time Base Generators Problems, Methods of Linearity improvement	L58	T2, R4	
TOTAL NO OF CLASSES 15					
TOTAL NO OF CLASSES 58					

(8) *SUGGESTED BOOKS*

TEXTBOOKS :

1. Integrated Electronics – Jacob Millman and Christos C Halkies, 1991 Ed. 2008, TMH.
2. Electronic Devices Conventional and Current Version – Thomas L. Floyd 2015, Pearson

REFERENCES :

1. Electronic Devices and Circuits, David A. Bell, 5.ed., Oxford University Press
2. Electronic Devices and Circuit Theory – Robet L. Boylestad, louis Nashelsky, 9.Ed., 2008 PE.
3. Introductory Electronic Devices and Circuits – Robert T. Paymer, 7.Ed., 2009, PEI.
4. Electronic Circuit Analysis – K. Lal Kishore, 2004, BSP.
5. Electronic Devices and Circuits – S. Salivahanan, N. Suresh Kumar, A Vallavaraj, 2.Ed., 2009, TMH
6. Microelectronics Circuits – Sedra and smith – 5.Ed., 2009, Oxford University.
7. Design of Analog CMOS Integrated Cicuits – Behzad Razavi, 2008, TMH.

(9) *WEBSITESFOR SELF LEARNING*

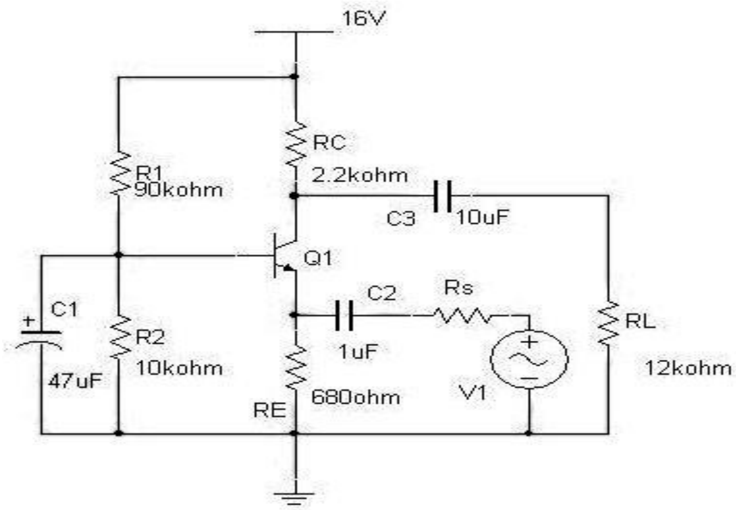
1. http://www.ni.com/multisim/what-is/
2. https://youtube.com/playlist?list=PLbRMhDVUMngehqNF2w_UbAi94qIycZOTG
3. http://www.mhhe.com/engcs/electrical/neamen01/
4. http://www.unix.eng.ua.edu/~huddl/mystuff/ECE333/ISM---Electronic%20Circuit%20Analysis%20and%20Design.pdf

5. http://www.stanford.edu/class/ee122/Handouts/EE113_Course_Notes_Rev0.pdf
6. http://www.ee.hacettepe.edu.tr/~solen/Matlab/MatLab/Matlab%20-%20Electronics%20and%20Circuit%20Analysis%20using%20Matlab.pdf
7. http://www.zen22142.zen.co.uk/Theory/tr_model.htm
8. http://en.wikipedia.org/wiki/Negative_feedback_amplifier
9. http://www.learnabout-electronics.org/Amplifiers/amplifiers10.php
10. http://www.electronics-tutorials.ws/oscillator/rc_oscillator.html
11. http://www.learnabout-electronics.org/Oscillators/osc10.php

(10.1) *QUESTION BANKS - JNTU*

1.a) For the CB amplifier circuit shown, compute R_{IN} and R_{OUT} if C_1 is
i) Connected ii) Not connected

The h-parameters of the transistor in CE configuration are listed as:
$h_{ie} = 2.1K\Omega$, $h_{fe} = 81$, $h_{oe} = 1.66\ \mu$Mhos and h_{re} is negligibly small.



b) Reason out the causes and results of Phase & Frequency distortions in transistor amplifiers.

2.a) Differentiate between direct and capacitive coupling of multiple stages of amplifiers.
b) With the help of a neat circuit diagram, describe the working of a cascode amplifier.
c) What are the merits and demerits of a cascade amplifier over a simple Common Emitter amplifier?

3.a) Derive the expressions for hybrid Π conductance, g_{ce}, and $g_{bb'}$ of a transistor.
b) Explain how hybrid Π parameters, g_m and g_{ce} vary with I_c, V_{ce} and temperature.
c) Compute the overall lower cut-off frequency of an identical two stage cascade of amplifiers with individual lower cut-off frequency given as 432 Hz.

4.a) Discuss the effect of different type of loads to a common source MOS amplifier.

b) Differentiate between cascode and folded cascode configurations.

5.a) If negative feedback with a feedback factor, β of 0.01 is introduced into an amplifier with a gain of 200 and bandwidth of 6 MHz, obtain the resulting bandwidth of the feedback amplifier.

b) With the help of a suitable BJT based voltage series feedback amplifier diagram, explain the feature and benefits of negative feedback amplifiers.

6.a) Substantiate the requirement of positive feedback in amplifier for oscillations. Relate the requirement to Barkhausen Criterion.

b) With the help of neat circuit diagram, explain how sustained oscillations are obtained in RC phase shift BJT based oscillator. Derive the expression for frequency of oscillations.

7.a) A single stage class A amplifier V_{cc}=20V, V_{CEQ} =10V, I_{CQ} =600mA, R_L=16 Ω. The ac output current varies by ± 300mA, with the ac input signal. Find

i) The power supplied by the dc source to the amplifier circuit.
ii) AC power consumed by the load resistor.
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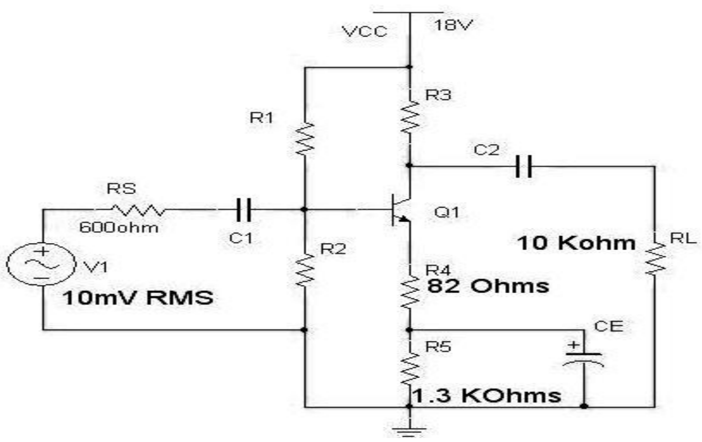
v) Overall efficiency
vi) Collector efficiency.

b). List the advantages of complementary-symmetry configuration over push pull configuration.

8. Describe the following briefly:

a) Stagger Tuned Amplifiers – Operation and comparison with synchronous tuning
b) Heat Sinks for tuned power amplifiers.

9. For the amplifier circuit shown with partially unbypassed emitter resistance, calculate the voltage gain with R_4 in place and with R_4 shorted. Consider h_{ie} = 1.1KΩ, h_{fe} = 100, h_{re} & h_{oe} are negligibly small. Assume R_1 and R_2 to be 100KΩ and 22 KΩ respectively.



b) Analyse what the output voltage should be if the DC power supply given to a CE amplifier is shorted to ground.

10..a) With the help of circuit diagram and equivalent circuit of a Darlington amplifier generate the expression for the overall input impedance of the pair.

b) Develop a generalized expression for overall current gain(A_{IS}) when two transistor stages with $R_{OUT2} < R_L$, $R_{OUT1} > R_{IN2}$, $R_{IN1} > R_S$ and individual voltage gains are A_{V1}, A_{V2}.

11.a) A transistor amplifier in CE configuration is operated at high frequency with the following specifications. f_T=6MHz, g_m=0.04,h_{fe} =50, $r_{bb'}$ =100 Ω, R_s =500 Ω , $C_{b'c}$ =10pF, R_L=100 Ω. Compute the voltage gain, upper 3dB cut-off frequency, and gain bandwidth product (GBW).

b) Derive an expression for the overall higher cut-off frequency of a two stage amplifier with identical stages of individual higher cut-off frequency, f_H.

12.a) Discuss the effect of different type of loads to a common source MOS amplifier.

b) Differentiate between cascode and folded cascode configurations.

13.a). If the non-linear distortion in a negative feedback amplifier with an open loop gain of 100 is reduced from 40% to 10%with feedback, compute the feedback factor, β of the amplifier.

b) Draw the circuit diagram of a current series feedback amplifier, Derive expressions to show the effect of negative feedback on input & output impedances, bandwidth, distortion of the amplifier.

14.a) Differentiate between RC and LC type oscillators.

b) Derive the expression for frequency of oscillation in a Hartley Oscillator.

c) State Barkhausen Critterion for Oscillations

15.a) Derive the expression for maximum conversion efficiency for a simple series fed Class A power amplifier.

b) What are the drawbacks of transformer coupled power amplifiers?

c) A push pull amplifier utilizes a transformer whose primary has a total of 160 turns and whose secondary has 40 turns. It must be capable of delivering 40W to an 8 Ω load under maximum power conditions. What is the minimum possible value of V_{cc}?

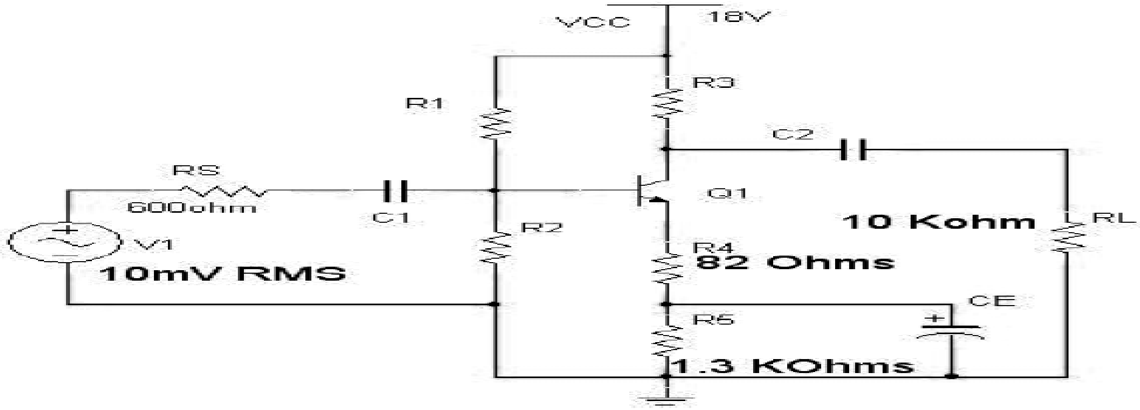
16.a) List possible configurations of tuned amplifiers.

b) Derive an expression for bandwidth of a capacitive coupled tuned amplifier in CE configuration. Make necessary assumptions and mention them.

17.a) For the common emitter amplifier shown, draw the AC and DC load lines. Determine the peak -to- peak output voltage for a sinusoidal input voltage of 30mV peak-to-peak. Assume C_1, C_2 and C_3 are large enough to act as short circuit at the input frequency. Consider h_{ie} = 1.1KΩ, h_{fe} = 100, h_{re} & h_{oe} are negligibly small.

b) State Miller's theorem. Specify its relevance in the analysis of a BJT amplifier.

c) Write expressions for A_V and R_{IN} of a Common Emitter amplifier.



18.a) Derive expressions for overall voltage gain and overall current gain of a two-stage RC coupled amplifier.

b) List out the special features of Darlington pair and cascode amplifiers.

19.a) Discuss the effect of emitter bypass capacitor and input & output coupling capacitors on the lower cut-off frequency if number of amplifiers are cascaded.

b) Describe how an emitter follower behaves at high frequencies.

20.a) Discuss the effect of different types of loads to a common source MOS amplifier.

b) Differentiate between cascode and folded cascode configurations.

21.a) The β and the open loop gain of an amplifier are -10% and -80 respectively. By how much % the closed loop gain changes if the open loop gain increases by 25%?

b) Compare the characteristics of feedback amplifiers in all the four configurations.

c) Reason out why 2 stages are required to implement current shunt feedback.

22. Starting from the description of a generalized oscillator, derive the expression for frequency of oscillation in a colpitts oscillator.

23.a) With the help of a suitable circuit diagram, show that the maximum conversion efficiency of a class B power amplifier is 78.5%.

b) Explain how Total harmonic distortion can be reduced in a Class B push-pull configured amplifier.

24.a) Derive an expression for the bandwidth of a synchronous tuned circuit.

b) Discuss the necessity of stabilization circuits in tuned amplifiers.

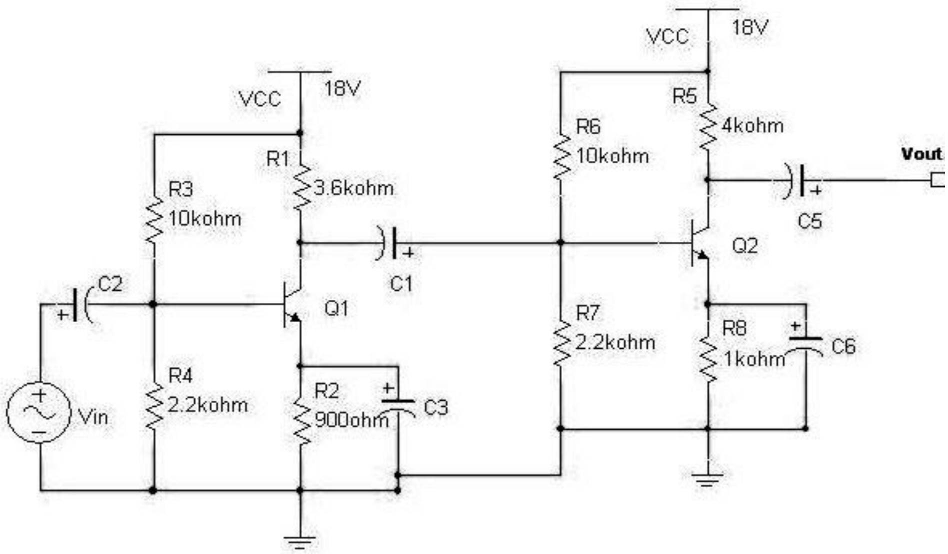
25.a) Draw the circuit diagram of a common collector amplifier along with its equivalent circuit. Derive expressions for A_V and R_I.

b) What is meant by small signal for analyzing a BJT based amplifier?

c) What is non-linear distortion? List the causes for this type of distortion in amplifiers.

26.a) Discuss various possibilities of inter-stage coupling of amplifiers.

b) For the two-stage RC coupled amplifier circuit shown, calculate the Individual stage voltage gains and the overall voltage gain. Input impedance of individual stages is given as 2.4 KΩ and β of individual transistors as 80.



27.a) A transistor has f_α = 8MHz, and β=80.when connected as an amplifier, it has stray capacitance of 100pF at the output terminal. Calculate its upper 3dB frequency when R_{load} is

i) 10KΩ ii) 100KΩ.

b) Discuss the effect of coupling capacitors of a CE amplifier on the overall frequency response of amplifier.

28.a) Discuss the effect of different type of loads to a common source MOS amplifier.

b) Differentiate between cascode and folded cascode configurations.

29.a) An amplifier has a gain of 50 with negative feedback. For a specified output voltage, if the input required is 0.1V without feedback and 0.8V with feedback, Compute β and open loop gain.

b) Through the block schematics, show four types of negative feedback in amplifiers.

c) List the advantages of negative feedback in amplifiers.

30.a) List out the merits × demerits of oscillators.

b) With the help of suitable schematic and description, show that both positive and negative feedback are used in a Wien Bridge oscillator. Establish the condition for oscillations.

31.a) State the merits of using push pull configuration? Describe the operation of class B push pull amplifier and show how even harmonics are eliminated.

b) A single ended class A amplifier has a transformer coupled load of 8 Ω. If the transformer turns ration is 10, find the maximum power output delivered to the load. Take the zero signal collector current of 500mA.

32.a) Derive the expressions for Bandwidth and Q-factor of single tuned, capacitively coupled amplifiers. List the assumptions made for the derivation.

b) What is stagger tuning? Suggest possible applications.

33.a) Draw the circuit diagram of Common Drain amplifier and derive an expression for its Voltage gain.

b) The h-parameters of the transistor used in CE amplifier are h_{fe} = 50, h_{ie} =1.1KΩ, h_{re} = $2.5x10^{-4}$, h_{oe} = 24μ A/V. Find out current gain and voltage gains with and without source resistance, input and output impedances, given that R_L = 10K and R_S = 1 K.

34.a) Discuss about different types of distortions that occur in amplifier circuits.

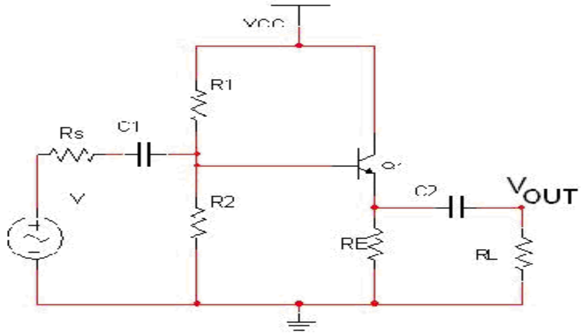
b) Three identical non interacting amplifier stages in cascade have an overall gain of 1 dB down at 30 Hz compared to mid band. Calculate the lower cutoff frequency of the individual stages.

35.a) Draw and explain the small signal equivalent circuit for an emitter follower stage at high frequencies.

b) Consider a CE stage with a resistive load R_L. Using Miller's theorem Find input capacitance at mid-band frequencies and high frequencies.

36. (a) For a single stage transistor amplifier, RS = 10K and RL= 10K. The h-parameter values are hfc = 51, hic =1.1K; hrc _ 1; hoc = 25 _A/V Find AI; AV; AV S, Ri, and Ro for the CC transistor configuration.
(b) For a single stage transistor amplifier, RS=1K; and RL= 10K the h-parameter values are hfe = 50, hie =1.1K; hre = 2.5x10□4, hoe = 25 _ A/V. Find AI; AV; AV S, Ri, and Ro for the CE transistor configuration.

37. (a) For the emitter follower circuit shown in _figure 1, calculate the quiescent voltage and current for VCC = 20Volts, hfc =120, hic = 1.1K, hoc = 2.5 x 10□6 mhos and hrc is negligibly Small. Reactance of capacitance need not be considered at the frequencies of interest.



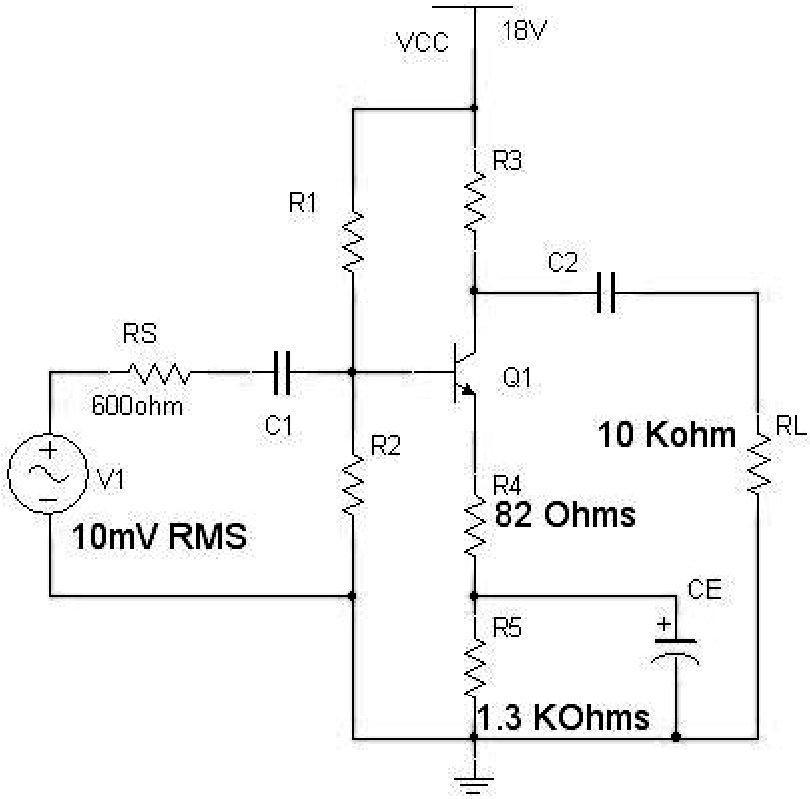
If R1 = 27K, R2 = 5.6K, RL = RE =220, RS = 0, Find the maximum undistorted peak-to-peak output voltage.
(b) Compare and contrast Common Emitter, Common Collector and Common Base amplifiers in all aspects.

38. (a) Derive the expression for the bandwidth of multistage amplifier.
(b) What are the problems of Direct coupled amplifiers?
(c) Why RC coupling is popular?
(d) Why transformer coupling is not used in the initial stage of a multistage amplifier?

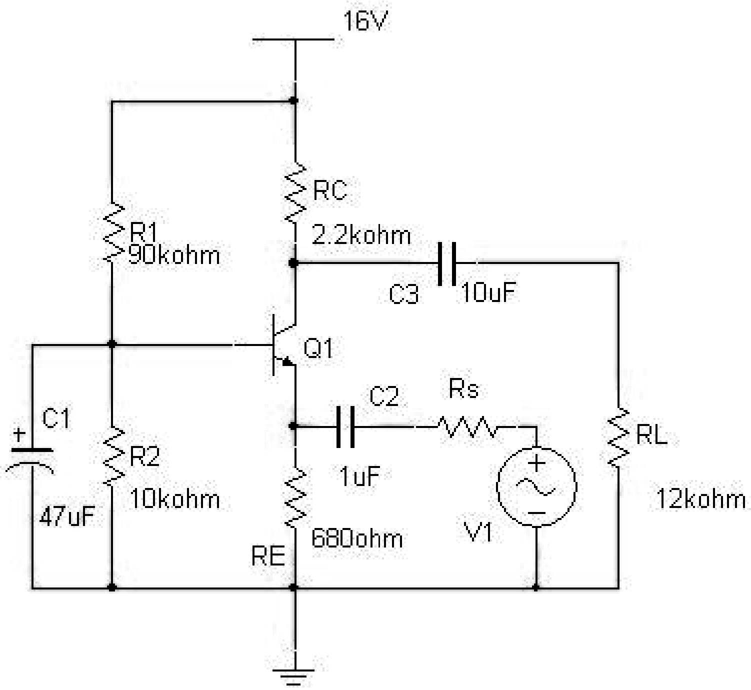
39. (a) Derive the expression for output conductance and diffusion capacitance of hybrid-_ CE amplifier.
(b) A single-stage CE ampli_er is to have a bandwidth fH of 5MHz with RL = 500. Assume hfe = 100, gm= 100mA/V, rbb0 = 100, Cc= 1PF, and fT =400 MHz
i. Find the value of the source resistance that will give the required band-width.
ii. with the value of Rs, determined in part (i), _nd the mid band voltage gain Vo / VS.

40.a) Differentiate between direct and capacitive coupling of multiple stages of amplifiers.
b) With the help of a neat circuit diagram, describe the working of a cascode amplifier.
c) What are the merits and demerits of a cascade amplifier over a simple Common Emitter amplifier?

41. For the amplifier circuit shown with partially unbypassed emitter resistance, calculate the voltage gain with R_4 in place and with R_4 shorted. Consider h_{ie} = 1.1KΩ, h_{fe} = 100, h_{re} & h_{oe} are negligibly small. Assume R_1 and R_2 to be 100KΩ and 22 KΩ respectively.



b) Analyse what the output voltage should be if the DC power supply given to a CE amplifier is shorted to ground.



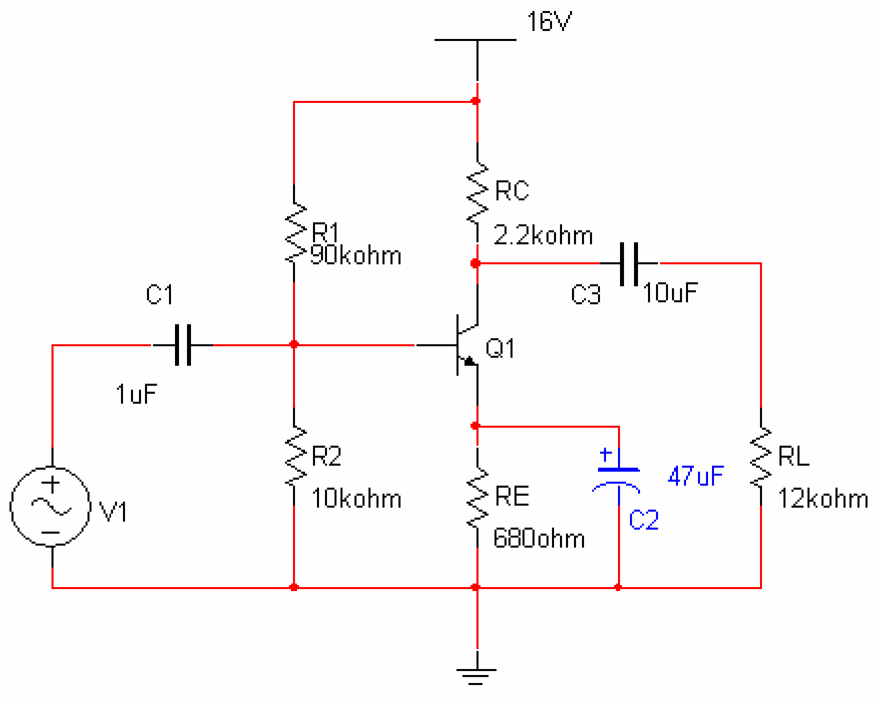
b) Reason out the causes and results of Phase & Frequency distortions in transistor amplifiers. [9+6] ha

b) State Miller's theorem. Specify its relevance in the analysis of a BJT amplifier.

c) Write expressions for A_V and R_{IN} of a Common Emitter amplifier.

42. Derive expressions for overall voltage gain and overall current gain of a two-stage RC coupled amplifier.

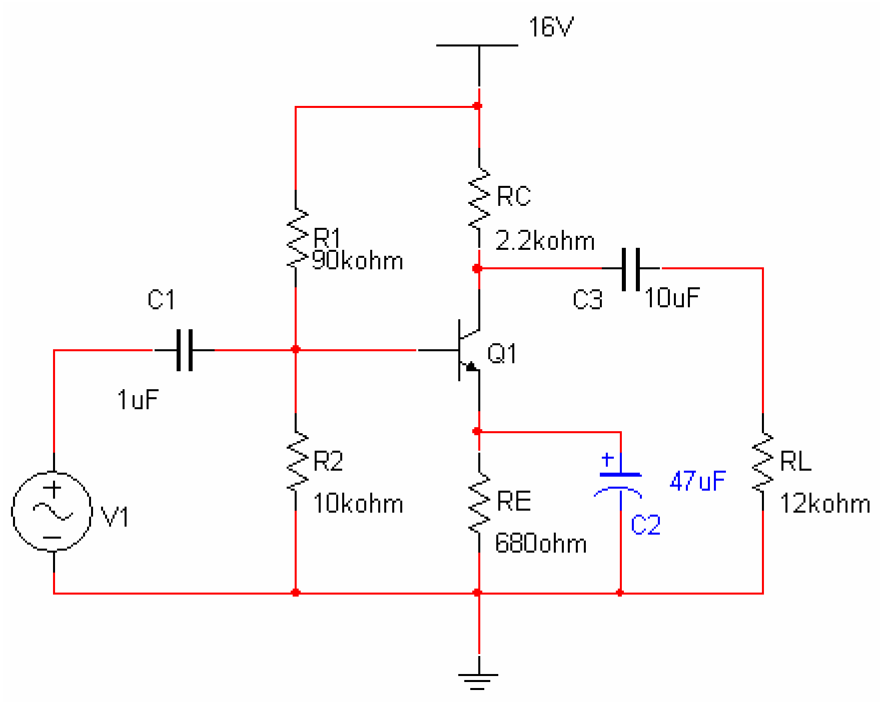
43.a) For the common emitter amplifier shown in Figure.1, Draw the AC and DC load lines. Determine the peak-to-peak output voltage for a sinusoidal input voltage of 30mV peak-to-peak. Assume C_1, C_2 and C_3 are large enough to act as short circuit at the input frequency. Consider h_{ie} = 1.1KΩ, h_{fe} = 100, h_{re} & h_{oe} are negligibly small.



b) List the merits and demerits of a JFET over a BJT.

c) Write the expressions for A_V and R_{in} of a CE amplifier.

44 a) for the common emitter amplifier shown in Figure.1, Draw the AC and DC load lines. Determine the peak-to-peak output voltage for a sinusoidal input voltage of 30mV peak-to-peak. Assume C_1, C_2 and C_3 are large enough to act as short circuit at the input frequency. Consider h_{ie} = 1.1KΩ, h_{fe} = 100, h_{re} & h_{oe} are negligibly small.



b) List the merits and demerits of a JFET over a BJT.
c) Write the expressions for A_V and R_{in} of a CE amplifier.

45.(a) Derive the expression for the high 3-dB frequency f _h of n-identical non inter-acting stages in terms of fH for one stage.
(b) If four identical amplifiers are cascaded each having fH =100 KHz, determine the overall upper 3dB frequency f _h. Assume non interacting stages.
(c) Write a short note on Bootstrapped Darlington circuit.

47. (a) Show that in Hybrid - model, the diffusion capacitance is proportional to the emitter bias current.
(b) What is the frequency range to consider Giacolletto model of a transistor at high frequencies? What is the significance of fT in discussing the frequency range of a transistor at high frequencies?

48. For a given CE amplifier with _=100, Ic=5mA, Vce=10V, hie=800, ho e = 1 0□4 mhos hre= 10□4. Take fT =50MHz and Cob=3pF. Compute all hybrid Q parameters.

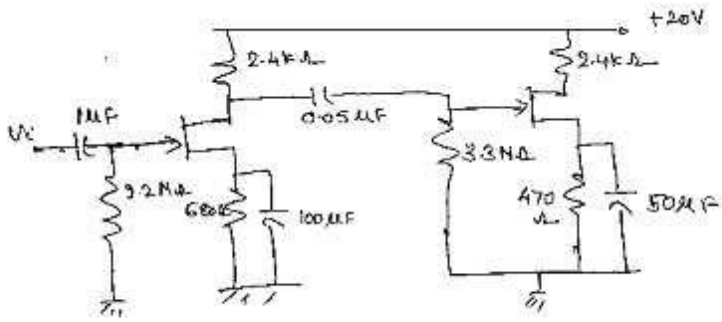
49. (a) Compare the inter-stage coupling methods in RC coupled amplifier and Darlington pair.
(b) Generate a generalized expression for overall current gain (AIS) when two transistor stages with ROUT2> RL, ROUT1 > RIN2 , RIN1> RS and individual voltage gains are AV 1 and AV 2 are cascaded.

50. (a) Draw the FET amplifier equivalent circuit looking into the drain and _nd its gain & o/p impedance?
(b) Starting with the definition of gm and rd, show that if two identical FETs are connected in parallel, gm is doubled and rd is halved since µ = rd gm, then remains unchanged.

51. (a) What is square law distortion? What is its effect in FET amplifiers? Compare

important characteristics of CD, CS, CG FET amplifier.
(b) A self-biased CE amplifier circuit has R1 = 100K , R2 = 10K, Rc = 5K, RE= 1K, RS = 10K. Compute AI , AV , AV S and Ri. The h-parameters of the transistor are hie = 1.1k, hfe = 50, hre = 2.5 x 10□4 , hoe = 25_A/V.

52. For the circuit shown, calculate the dc bias, voltage gain, input impedance and output impedance for the cascade amplifier shown in Figure.



53.a) With the help of circuit diagram and equivalent circuit of a Darlington amplifier generate the expression for the overall input impedance of the pair.
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$R_{OUT2} < R_L$, $R_{OUT1} > R_{IN2}$, $R_{IN1} > R_S$ and individual voltage gains are A_{V1}, A_{V2}.

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b) Derive an expression for the overall higher cut-off frequency of a two stage amplifier with identical stages of individual higher cut-off frequency, f_H.
55.a) Discuss the effect of different type of loads to a common source MOS amplifier.
b) Differentiate between cascode and folded cascode configurations.
b) List out the special features of Darlington pair and cascode amplifiers.

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b) List out various types of distortions that occur in transistor amplifiers. Discuss the causes for each type listed.

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60. (a) Draw the circuit of a voltage series feedback circuit and explain it.
(b) What are the possible amplifiers circuits in any feedback system? Discuss.

61. Obtain the expressions for the voltage gain in the low frequency, medium frequency and high frequency ranges in the case of single stage amplifier.

62. (a) Give the two Barkhausen conditions required in order for sinusoidal oscillations to be sustained.
(b) Draw the circuit diagram of RC phase - shift oscillator and derive the expression for frequency of Oscillations & condition for sustained Oscillations.

63.a) If negative feedback with a feedback factor, β of 0.01 is introduced into an amplifier with a gain of 200 and bandwidth of 6 MHz, obtain the resulting bandwidth of the feedback amplifier.
b) With the help of a suitable BJT based voltage series feedback amplifier diagram, explain the features and benefits of negative feedback in amplifiers.

64.a). If the non-linear distortion in a negative feedback amplifier with an open loop gain of 100 is reduced from 40% to 10%with feedback, compute the feedback factor, β of the amplifier.
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b) Compare the characteristics of feedback amplifiers in all the four configurations.
c) Reason out why 2 stages are required to implement current shunt feedback.

66. Starting from the description of a generalized oscillator, derive the expression for frequency of oscillation in a colpitts oscillator.

67.a) What is Harmonic distortion in transistor amplifier circuits? Discuss on second harmonic distortion.
b) Draw and explain the operation of Class-AB power amplifier. How will it eliminate cross over distortion?

68.a) Draw and explain the circuit diagram of a single tuned Capacitance coupled amplifier. Also explain its operation.
b) Draw and explain the significance of Gain versus Frequency curve of tuned amplifiers when they are used in radio receivers.
c) Draw the Ideal and actual frequency response curves of a single tuned amplifier.

69. Explain in detail the effect of cascading tuned amplifiers and hence derive the expression for bandwidth of n-stage amplifier. Also draw the frequency response and explain what happens as the number of stages increases?

70. (a) Draw the circuit of class -A series fed power amplifier and derive the expression for output power Po. (b) Draw and discuss the operation of Class - C power amplifier.

71.(a) What is coefficient of coupling in a double tuned amplifier? Discuss its effect on the frequency response.
(b) Derive the expression for bandwidth of a double tuned amplifier.

72. (a) A transistor supplies 0.85W to a 5K load, the zero signal dc collector current is 30mA, and the dc collector current with signal is 36mA.Determine the percent harmonic distortion.
(b) Derive an expression for conversion efficiency of a class B power amplifier.

73. (a) Why two tuned circuits are used in double tuned amplifier?
(b) What are the advantages of stagger tuned amplifier?
(c) Why parallel resonance circuits are used in tuned amplifiers?

74. (a) A single ended class A power amplifier is coupled to an 8 load, using a transformer with a turn ratio of 5:1 with a 50V supply the transistor is biased to have a quiescent collector current of 250mA. When a sinusoidal signal is applied to the base, the collector voltage varies between a maximum of 5V and maximum of 90V. Estimate the efficiency, power output & second – harmonic distortion of this stage.
(b) Discuss how rectification may takes place in a power amplifier?

75. (a) Explain the differences between the function of a transformer used in a power amplifier and that used in a double tuned voltage amplifier.
(b) Explain the method of determination of total harmonic distortion in push pull power amplifiers using 5 - point analysis.

(c) Calculate the harmonic distortion components for an output signal, in push pull power amplifiers having fundamental amplitude of 2.5 Volts, second harmonic amplitude of 0.25 Volts, third harmonic amplitude of 0.1 Volts, fourth harmonic amplitude of 0.05V. Also calculate the total harmonic distortion.

76. (a) Determine the input power, output power and efficiency for a class B power amplifier circuit with Vcc=30 V , Im=1 Amp and RL=10 .
(b) Draw the circuit of transformer less pushpull amplifier circuit with loud speaker as the load resistance. Justify the circuit behavior with \emitter follower" circuit operation.

77.a) Substantiate the requirement of positive feedback in amplifier for oscillations. Relate the requirement to Barkhausen Criterion.
b) With the help of neat circuit diagram, explain how sustained oscillations are obtained in RC phase shift BJT based oscillator. Derive the expression for frequency of oscillation.

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iii) AC power developed across the load resistor.
iv) DC power wasted in transistor collector.
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b). List the advantages of complementary-symmetry configuration over push pull configuration

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a) Stagger Tuned Amplifiers – Operation and comparison with synchronous tuning
b) Heat Sinks for tuned power amplifiers.

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82.a) List possible configurations of tuned amplifiers.
b) Derive an expression for bandwidth of a capacitive coupled tuned amplifier in CE configuration. Make necessary assumptions and mention them.

83.a) With the help of a suitable circuit diagram, show that the maximum conversion efficiency of a class B power amplifier is 78.5%.
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b) A single stage class A amplifier V_{cc}=20V, V_{CEQ}=10V, I_{CQ}=600mA, R_L=16 Ω. The ac output current varies by ±300mA, with the ac input signal. Find
i) Power supplied by the dc source to the amplifier circuit.
ii) The AC power consumed by the load resistor.
iii) Conversion efficiency.

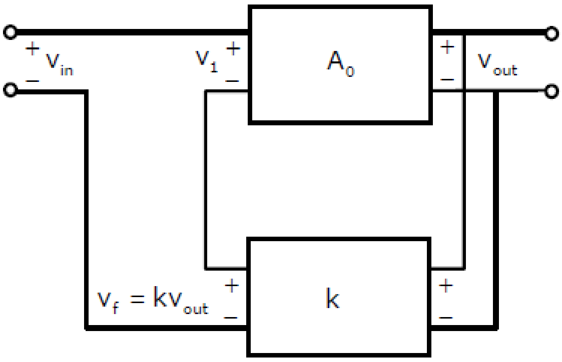
86. a)Derive an expression for 'bandwidth' and 'quality factor' of a capacitive coupled single tuned amplifier in CE configuration. Make necessary assumptions and mention them.
b). Substantiate the necessity of the following in tuned amplifiers.
a) Heat Sinks
b) Stabilization circuits

87.a) Derive the expression for maximum conversion efficiency for a simple series fed Class A power amplifier. What are the drawbacks of transformer coupled power amplifiers?
b) A single stage class A amplifier V_{cc}=20V, V_{CEQ}=10V, I_{CQ}=600mA, R_L=16 Ω. The ac output current varies by ±300mA, with the ac input signal. Find
i) Power supplied by the dc source to the amplifier circuit.
ii) The AC power consumed by the load resistor.
iii) Conversion efficiency.

(10.2) ***QUESTION BANK – GATE***

1. In a voltage-voltage feedback as shown below, which one of the following statements is TRUE if the gain k is increased?

GATE 2013



(A) The input impedance increases and output impedance decreases
(B) The input impedance increases and output impedance also increases
(C) The input impedance decreases and output impedance also decreases
(D) The input impedance decrases and output impedance increases

2. The ac schematic of an NMOS common-source stage is shown in the figure below, where part of the biasing circuits has been omitted for simplicity. For the nchannel MOSFET M, the Transconductance m g □□1mA / V, and body effect and channel length modulation effect are to be neglected. The lower cutoff frequency in Hz of the circuit is approximately at

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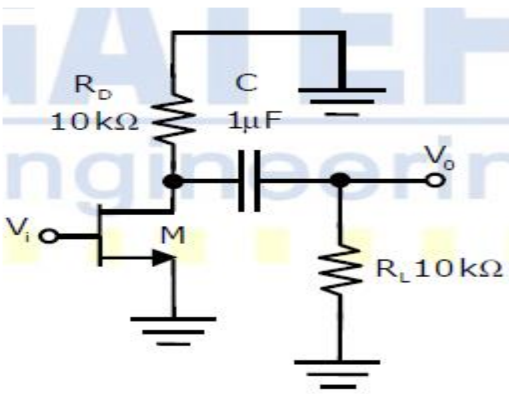

(A) 8 (B) 32 (C) 50 (D) 200

3. The current *ib* through the base of a silicon *npn* transistor is 1 + 0.1 cos (10000*πt*)mA At 300 K, the *rπ* in the small signal model of the transistor is

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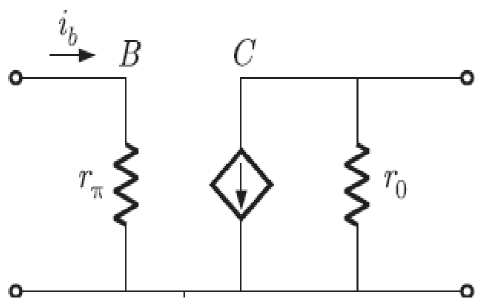

(A) 250 Ω (B) 27.5 Ω
(C) 25 Ω (D) 22.5 Ω

4. The voltage gain *Av* of the circuit shown below is

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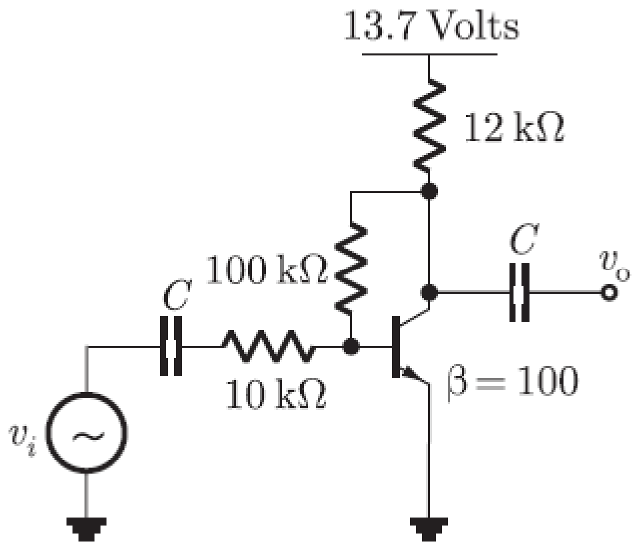
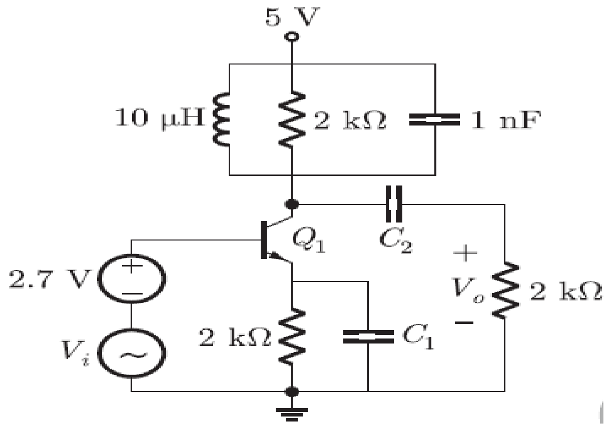


(A) *Av* = 200 (B) *Av* = 100
(C) *Av* = 20 (D) *Av* = 10

5. In the circuit shown below, capacitors *C*1 and *C*2 are very large and are shorts at the input frequency. *vi* is a small signal input. The gain magnitude Vo/Vi at 10 M rad/s is

GATE 2012

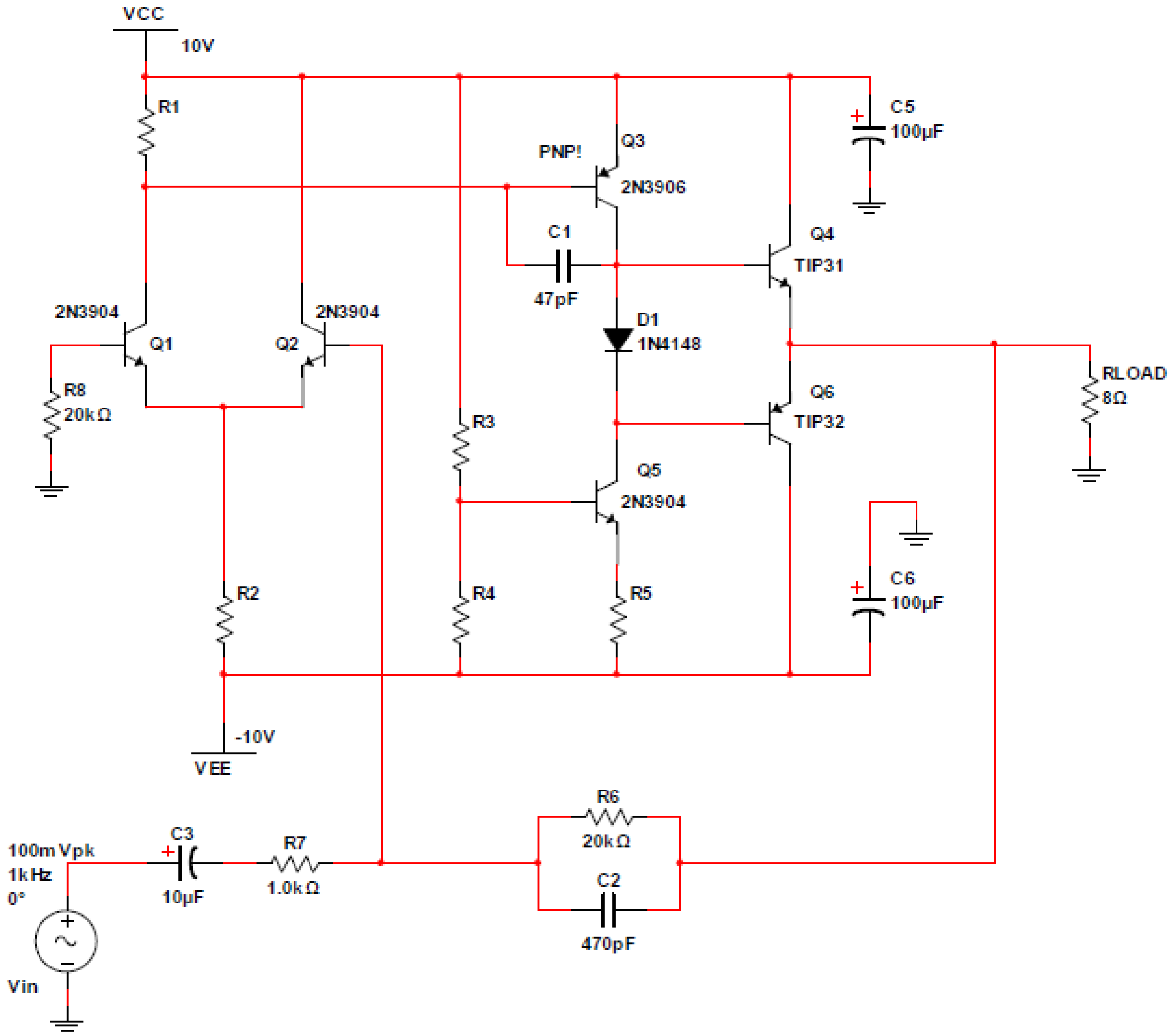


(A) maximum (B) minimum (C) unity (D) zero

(11) *TWO CASE STUDY PRESENTATIONS*

1. A 3-STAGE 5W AUDIO AMPLIFIER:

A 3-stage audio amplifier utilizing three BJT amplifier building blocks – a Differential Pair, a Common-Emitter Amplifier (with active current source load) and an Emitter Follower is shown below. Each stage is biased by a constant current source, and a feedback network is used to set the overall gain of the amplifier. The input signal ***vin*** represents any line-level audio signal generated by a function generator, guitar pick-up or mp3 player. The resistive load represents an 8Ω audio speaker.



The circuit is configured as an *inverting* amplifier with a gain of $AV = -$ R6/R7 or -20 V/V in the pass band. The impedance network comprised of R6, C2, R7, C3 provides a band-pass filter response with cutoff frequencies of approximately 16Hz and 16kHz, covering most of the audio frequency range. C1 provides *dominant pole compensation* to stabilize the closed-loop frequency response of the amplifier. Without this capacitor, the circuit may oscillate. The output stage has been modified to include an Emitter Follower Push-Pull Pair. By adding the complementary PNP power transistor, the circuit no longer wastes the 1A bias current required if only one Emitter Follower is used. Although this modification saves power, it introduces *crossover distortion* since the transistors turn off when the signal voltage is below +/- 0.7 volts. D1 has been added in the second stage to provide a small DC offset when driving the Push-Pull Pair. This helps reduce cross-over distortion. Two series diodes would reduce distortion further BUT this would increase the risk of turning on both power transistors at the same time and shorting out the power supplies. Capacitors C5 and C6 have been added to filter the power supply locally near each power transistor. This helps reduce power supply bounce when the power transistors call for high current.

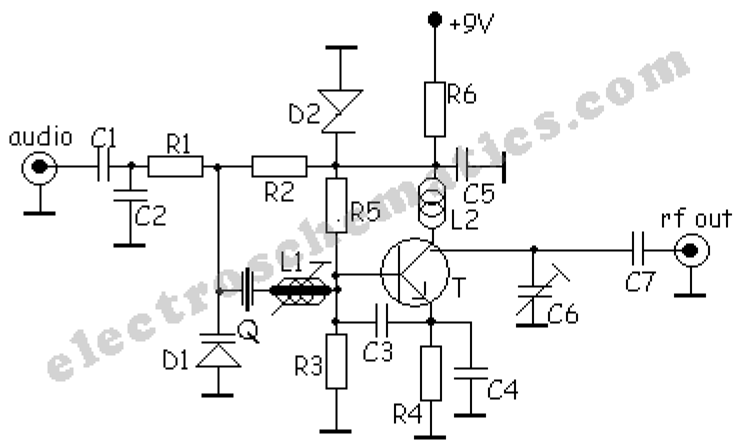
<u>2) Crystal Oscillator with 455 KHZ FREQUENCY</u>

Another great crystal oscillator circuit with BB139, a 2-nd harmonic quartz crystal and one BF214 transistor. So, the Q quartz is half of the desired transmitter frequency, L1 has 24 turns, 0.08mm Ø built on a 455kHz carcase. L2 has 8-9 turns / 0.8mm Ø / 5mm Ø. This oscillator circuit has a great frequency stability,

Crystal oscillator circuit components

R1 = 10K
R2 = 100K
R3 = 8.2K
R4 = R6 = 820Ω
R5 = 15K
C1 = 330nF
C2 = 1nF
C3 = 1.5pF
C4 = 15pF
C5 = 2.2nF
C6 = 6 – 25pF trimmer
C7 = 4.7Pf

D1 = BB139
D2 = 5.6v zenner diode
T = BF214
Q = half frequency quartz crystal



(12) ***ASSIGNMENT QUESTION/ INNOVATIVE ASSIGNMENT SETS***

Unit – I

SET 1:

1. Obtain the expressions for the voltage gain in the low frequency, medium frequency and high frequency ranges in case of single stage amplifier.
2. Explain the Principle of opeartion of direct coupled ampli er and mention its advantages.
3. What is the use of transformer coupling in the output stage of multi stage amplier?
4. Why RC coupling is mostly used for voltage amplier.

SET 2:

1. Define unity gain frequency. Obtain the necessary relation using transistor frequency response.
2. Differentiate between direct and capacitive coupling of multiple stages of amplifiers.
3. With the help of a neat circuit diagram, describe the working of a cascode amplifier.
4. What are the merits and demerits of a cascade amplifier over a simple Common Emitter amplifier.

SET 3:

1. With the help of circuit diagram and equivalent circuit of a Darlington amplifier generate the expression for the overall input impedance of the pair.
2. Develop a generalized expression for overall current gain(A_{IS}) when two transistor stages with $R_{OUT2} < R_L$, $R_{OUT1} > R_{IN2}$, $R_{IN1} > R_S$ and individual voltage gains are A_{v1}, A_{v2}
3. Derive expressions for overall voltage gain and overall current gain of a two-stage RC coupled amplifier.
4. List out the special features of Darlington pair and cascode amplifiers.

SET 4:

1. Discuss various possibilities of inter-stage coupling of amplifiers.
2. Write the equation for overall gain of a n - stage cascaded Ampli er.

3. How does the frequency response an ampli er change with cascading of ampli er stages?

4. Explain the choice of con guration in a cascade of ampliers. 5.

Unit - II

SET 1:

1. Draw the circuit of a voltage series feedback circuit and explain it.

2. What are the possible ampli ers circuits in any feedback system? Discuss.

3. Draw a feedback ampli er in block diagram form and explain each block giving its function.
4. Distinguish between regenerative and degenerative feedback in ampliers.

SET 2:

1. Draw the equivalent circuit for a current ampli er and what are the values of R_i & R_o for ideal ampli er?

2. The open loop gain of an ampli er is 100. What will be the overall gain when a negative feed back of 0.5 is applied to the ampli er?

3. What are the di erent mixing techniques used in any feed back system? Explain.

4. State the condition in terms of (1 + A) which a feed back ampli er must satisfy in order to be stable.

SET 3:

1. If negative feedback with a feedback factor, β of 0.01 is introduced into an amplifier with a gain of 200 and bandwidth of 6 MHz, obtain the resulting bandwidth of the feedback amplifier.

2. With the help of a suitable BJT based voltage series feedback amplifier diagram, explain the features and benefits of negative feedback in amplifiers.
3. If the non-linear distortion in a negative feedback amplifier with an open loop gain of 100 is reduced from 40% to 10%with feedback, compute the feedback factor, β of the amplifier.

4. Draw the circuit diagram of a current series feedback amplifier, Derive expressions to show the effect of negative feedback on input & output impedances, bandwidth, distortion of the amplifier.

SET 4:

1. Through the block schematics, show four types of negative feedback in amplifiers.

2. List the advantages of negative feedback in amplifiers.
3. Derive an expression for the transfer gain of a feedback amplier.

4. Discuss about the types of negative feedback ampli ers giving the effect of each type of feedback on the parameters of the amplier.

Unit - III

SET 1:

1. Give the two Barkhausen conditions required in order for sinusoidal oscillations to be sustained.

2. Draw the circuit diagram of RC phase - shift oscillator and derive the expression for frequency of Oscillations & condition for sustained Oscillations.

3. In a colpitts oscillator, C_1 = 0.2 F and C_2 = 0.04 F. If the frequency of oscillation is 10KHz, nd the value if Inductor. Also, nd the required gain for oscillation.

4. Determine the frequency of oscillations in a wien bridge oscillator.

SET 2:

1. Draw the colpilts oscillator circuit and explain its working.
2. Substantiate the requirement of positive feedback in amplifier for oscillations. Relate the requirement to Barkhausen Criterion.

3. With the help of neat circuit diagram, explain how sustained oscillations are obtained in RC phase shift BJT based oscillator. Derive the expression for frequency of oscillation.

4. Differentiate between RC and LC type oscillators.

SET 3:

1. Derive the expression for frequency of oscillation in a Hartley Oscillator.

2. State Barkhausen Critterion for Oscillations
3. Starting from the description of a generalized oscillator, derive the expression for frequency of oscillation in a colpitts oscillator.
4. List out the merits & demerits of oscillators.

SET 4:

1. Draw the electrical model of a piezoelectric crystal.

2. Over what portion of the reactance curve do we desire oscillations to take place when the crystal is used as part of a sinusoidal oscillator? Explain.

3. Sketch a circuit of a crystal - controlled oscillator and explain its function.

4. Explain the frequency - stability criterion for a sinusoidal oscillator.

Unit –IV
LARGE SIGNAL AMPLIFIERS

SET 1:

1. Show that the maximum conversion efficiency of the idealized class B push - pull circuit is 78.5%.
2. Explain the origin of crossover distortion. Describe a method to minimize this distortion.

3. What is class B amplifier? Why is it employed? Give its circuits, design equations, characteristics & limitations.

4. A transformer coupled Class A large signal amplifier has maximum and minimum values of collector to emitter voltage of 25V and 2.5V. Determine its collector efficiency.

SET 2:

1. A single stage class A amplifier V_{cc}=20V, V_{CEQ} =10V, I_{CQ} =600mA, R_L=16 Ω. The ac output current varies by ± 300mA, with the ac input signal. Find

    i) The power supplied by the dc source to the amplifier circuit.
    ii) AC power consumed by the load resistor.
    iii) AC power developed across the load resistor.
    iv) DC power wasted in transistor collector.
    v) Overall efficiency
    vi) Collector efficiency.

2. List the advantages of complementary-symmetry configuration over push pull configuration.

3. Derive the expression for maximum conversion efficiency for a simple series fed Class A power amplifier.

4. What are the drawbacks of transformer coupled power amplifiers?

SET 3:

1. With the help of a suitable circuit diagram, show that the maximum conversion efficiency of a class B power amplifier is 78.5%.

2. Explain how Total harmonic distortion can be reduced in a Class B push-pull amplifier.

3. State the merits of using push pull configuration? Describe the operation of class B push pull amplifier and show how even harmonics are eliminated.

4. A single ended class A amplifier has a transformer coupled load of 8 Ω. If the transformer turns ration is 10, find the maximum power output delivered to the load. Take zero signal collector current of 500mA.

SET 4:

1. What is push-pull configuration and how does this circuit reduce the harmonic distortion?

2. Derive an expression for the output power of a class A large - signal amplifier in terms of Vmax, V_{min} I_{max} & I_{min}.

3. What is a class B amplifier? Where is it employed? Give its circuits, design equations, characteristics & limitations.

4. A transistor supplies 0.8W to a 5K load. The zero signal dc collector current is 30mA, and the dc collector current with signal is 36mA. Determine the percent second - harmonic distortion.

TUNED AMPLIFIERS

SET 1:

1. Why two tuned circuits are used in double tuned amplifier?

2. What the advantages are of stagger tuned amplifier?

3. Compare neutralization and unilaterlization methods of tuned amplifiers

4. What the limitations are of stagger tuned amplifiers?

SET 2:

1. What happen when no. of stages is increased in single tuned cascaded amplifiers

2. How the frequency response of doubled tuned amplifier depends on degree of coupling between two tank circuits?

3. Draw the equivalent circuit of double tuned amplifier and derive the expression for gain at resonance.

4. Derive the expression for effective bandwidth of cascaded tuned amplifier.

SET 3:

1. Describe briefly Stagger Tuned Amplifiers – Operation and comparison with synchronous tuning
2. List possible configurations of tuned amplifiers.
3. Derive an expression for bandwidth of a capacitive coupled tuned amplifier in CE configuration.
4. What is stagger tuning? Suggest possible applications.

SET 4:

1. What the limitations are of stagger tuned amplifiers?
2. List possible configurations of tuned amplifiers.
3. Derive the expression for effective bandwidth of cascaded tuned amplifier.
4. Why two tuned circuits are used in double tuned amplifier?

UNIT V

SET-1

1(a) With reference to multivibrators, explain: i) stable-state ii) loop-gain iii) quasi stable-state

(b) Describe multivibrators from the viewpoints of construction, principle of working, classification
based on the output states, applications and specifications. Mention one specific application of each.

2. Draw and explain the circuit of Astable Multivibrator with necessary waveforms and also derive the
Expression for its frequency of oscillations.

3. (a) What is meant by triggered sweep? What are the merits and demerits of triggered sweep circuits?
(b) What is relaxation oscillator? Name some negative resistance devices used in relaxation oscillators and
give its applications

4. Explain the principle of working of Miller sweep circuit. Derive the expression for sweep speed
by taking Miller integrator circuit.

SET-2

1. Explain the operation of emitter-coupled bistable multivibrator. Also discuss different methods of
triggering a bistable multivibrator.

2. What is dead-band in a Schmitt trigger? Draw the hysteresis loop and explain how hysteresis can be
eliminated in a Schmitt trigger.

3. (a) What is a linear time base generator?
   (b) Write the applications of time base generators.
   (c) Define the sweep speed error, displacement error and transmission error of voltage time base Waveform
4. (a) Draw the circuit of a Boot strap sweep generator and explain its operation. Derive an expression for its sweep time.
   (b) Explain with a circuit the working of a UJT sweep circuit and obtain the expressions for the intrinsic standoff ratio (η).

SET-3

1. (a) Explain the reason for the occurrence of overshoot at the base of normally ON transistor of one shot. Derive an expression for overshoot.
   (b) Discuss a few applications of a monostable multivibrator. Explain how it differs with that of a binary.
2. (a) Draw the equivalent circuit diagram for differential amplifier used as an astable multi vibrator.
   (b) Draw the various waveforms at base and collector of transistors of the astable multivibrator
3. (a) Briefly describe various methods to achieve sweep linearity in time-base circuits.
   (b) Draw the circuit of a constant current sweep circuit, explain its operation and derive the expression for sweep voltage.
4 Draw a bootstrap sweep circuit using Darlington pair. Explain its operation with neat waveforms also mention its merits and limitations

SET-4

1. What is a monostable multivibrator? Explain with the help of a neat circuit diagram the principle of operation of a monostable multi, and derive an expression for pulse width. Draw the wave forms at collector and Bases of both transistors
2.(a) Define sweep speed error, transmission error and displacement error pertaining to sweep circuits. Also derive the expressions for the same with respect to an exponential sweep circuit
   (b) Explain the operation of an emitter coupled monostable multivibrator.
3. (a)Define the three errors that occur in a sweep circuit and obtain an expression for these errors for an exponential sweep circuit
   (b)What are the essential requirements of TV horizontal sweep circuit? How do you achieve them using a current sweep?
4. (a) With neat sketches and necessary expressions, explain the transistor Miller time-base generator

(b) Compare the principle of operation of Miller sweep circuit and Bootstrap sweep circuit

Innovative Assignment Questions

Subject:Electronic Circuit Analysis

SET-1

1) Design and Analysis of Multistage Common Emitter Amplifier for Low Frequency Applications?

2) Design Three Stage Amplifier with Current Limiter?

3) Design and Simulate 5 watts Audio Amplifier using Multisim?

4)Explain how the microphone in Amplifier works.How Exactly does it convert Waves of air Pressure into Electrical Signals?

5)The Voltage Gain of an Amplifier without feedback is 3000.Calculate the Voltage gain of the Amplifier if negative voltage Feedback is introduced in the circuit. Given that Feed back factor is 0.01.

SET-2

1) Design Three Stage Amplifiers with Current Limiter?

2) Design and Simulate 5 watts Audio Amplifier using Multisim?

3) Design and Analysis of Multistage Common Emitter Amplifier for Low Frequency Applications?

4) Explain how the microphone in Amplifier works. How exactly does it convert Waves of air Pressure into Electrical Signals?

5) The Voltage Gain of an Amplifier without feedback is 3000.Calculate the Voltage gain of the Amplifier if negative voltage Feedback is introduced in the circuit. Given that Feed back factor is 0.01.

SET-3

1) Design and Analysis of Multistage Common Emitter Amplifier for Low Frequency Applications?

2) Explain how the microphone in Amplifier works. How exactly does it convert Waves of air Pressure into Electrical Signals?

3)The Voltage Gain of an Amplifier without feedback is 3000.Calculate the Voltage gain of the Amplifier if negative voltage Feedback is introduced in the circuit. Given that Feed back factor is 0.01.

4) Design Three Stage Amplifiers with Current Limiter?

5) Design and Simulate 5 watts Audio Amplifier using Multisim?

(13) *LIST OF TOPICS FOR STUDENT'S SEMINARS*

1. Magnetic Amplifiers
2. Oven Controlled Crystal Oscillators
3. Crystal Oscillators for Industrial Applications
4. Stereo audio amplifier with digital volume control
5. Adjustable High/Low Frequency Sine wave generator
6. Nostalgic Crystal Radio
7. MSF Radio Time Clock
8. CB receiver
9. Band-Pass and Notch Filters
10. CB transmitter

(14) *STEP/COURSE MATERIAL*

ECA Material.rar

(15) *EXPERT LECTURES WITH TOPICS & SCHEDULES*

1. Guest lecture on "Design and Analysis of Multistage amplifiers, Feedback amplifiers and Oscillators" tentatively scheduled in between 26/04/2023 and 30/04/2023.

□□□□***THE END*** □□□

ACADEMIC PLAN

FOR

ACADEMIC YEAR

2023-24

COURSE: II YEAR B.TECH ECE-II-SEM-R22

SUBJECT: Electromagnetic Fields and Transmission Lines

CREDITS: 3

ACADEMIC PLANNER
Subject: Electromagnetic Fields and Waves

S.NO **CONTENT**

(1) - Preamble/Introduction
(2) - Prerequisites
(3) - Objectives and Outcomes
(4) - Syllabus
1. JNTU/R20-CMREC
2. GATE
3. IES
(5) - List of Expert Details (Local/National/International with Contact details/Profile link/Blogs/their research Contribution towards the subject)
(6) - Journals with min 5 ref paper for literature study
(7) - Subject -Lesson plan
(8) - Suggested Books (prescribed and References)
(9) - Websites for self learning Resources like (Coursera, NPTEL, MIT open course etc)
(10) - Question Banks
1.JNTUH/Model papers
2. GATE
(11) - Two case study presentations with Project / Product/ Model /prototypes/ Industrial applications.
(12) - Assignment Question/Innovative Assignments sets.
(13) - List of topics for students Seminars with Guidelines
(14) - STEP/Course material in softcopy
(15) - Expert Lectures with topics & Schedules (if any)

(1) PREAMBLE/INTRODUCTION:

EM is all around us. In simple terms, every time we turn a power switch on, every time we press a key on our computer keyboard, or every time we perform a similar action involving an everyday electrical device, EM comes into play. It is the foundation for the technologies of electrical and computer engineering, spanning the entire electromagnetic spectrum, from dc to light, from the electrically and magnetically based (electro-mechanics) technologies to the electronics technologies to the photonics technologies.

(2) PREREQUISITES:

- Mathematics

(3) COURSE OBJECTIVE AND OUTCOMES:

Objectives:

- To learn Basic Laws, Concepts and Proofs related to electrostatic fields and magneto static fields, and apply them to solve physics and engineering problems
- To distinguish between static and time varying fields and understand the significance and utility of Maxwell's equations and boundary conditions and gain ability to provide solutions to communication engineering problems.
- To Study the propagation ,reflection and transmission of plane waves in bounded and unbounded media.

Outcomes:

After Completing this course, the students would be able to

- Acquire the knowledge of basic laws and concepts and proofs related to Electrostatic fields and magneto static fields.
- Characterize the static and time varying fields; establish the corresponding sets of Maxwell's equations and boundary conditions.
- Analyze the wave equations and classify conductors, dielectrics and evaluate the UPW characteristics for several practical media of interest.
- Analyze the Design aspect of transmission line parameters and configuration.

(4) SYLLABUS:

Electromagnetic Fields and Transmission Lines

II YEAR B.TECH ECE-II SEM **L T P C**

3 0 0 3

UNIT – I

Electrostatics: Coulomb's Law, Electric Field Intensity – Fields due to Different Charge Distributions, Electric Flux Density, Gauss Law and Applications, Electric Potential, Relations Between E and V, Maxwell's Two Equations for Electrostatic Fields, Energy Density, Illustrative Problems. Convection and Conduction Currents, Dielectric Constant, Isotropic and Homogeneous Dielectrics, Continuity Equation, Relaxation Time, Poisson's and Laplace's Equations; Capacitance – Parallel Plate, Coaxial, Spherical Capacitors,

UNIT – II

Magneto statics: Biot-Savart's Law, Ampere's Circuital Law and Applications, Magnetic Flux Density, Maxwell's Two Equations for Magneto static Fields, Magnetic Scalar and Vector Potentials, Forces due to Magnetic Fields, Ampere's Force Law.

UNIT – III

Maxwell's Equations (Time Varying Fields): Faraday's Law and Transformer EMF, Inconsistency of Ampere's Law and Displacement Current Density, Maxwell's Two equations for magnetostatics, Maxwell's Two equations for electrostatics Fields, Maxwell's Equations in Different Forms, Conditions at a Boundary Surface : Dielectric-Dielectric and Dielectric-Conductor Interfaces.

UNIT – IV

EM Wave Characteristics: Wave Equations for Conducting and Perfect Dielectric Media, Uniform Plane Waves – Definition, All Relations Between E & H, Sinusoidal Variations, Wave Propagation in Lossless and Conducting Media, Conductors & Dielectrics – Characterization, Wave Propagation in Good Conductors and Good Dielectrics, Polarization.

Reflection and Refraction of Plane Waves – Normal and Oblique Incidences for both Perfect Conductor and Perfect Dielectrics, Brewster Angle, Critical Angle and Total

Internal Reflection, Surface Impedance, Poynting Vector and Poynting Theorem – Applications.

UNIT – V

Transmission Lines: Types, Parameters, Transmission Line Equations, Primary & Secondary Constants, Expressions for Characteristics Impedance , Propagation Constant, Phase and Group Velocities, Infinite Line Concepts, Losslessness/Low Loss Characterization, Distortion - Condition for Distortionlessness and Minimum Attenuation, Loading - Types of Loading.

Input Impedance Relations, SC and OC Lines, Reflection Coefficient, VSWR. UHF Lines as Circuits Elements; λ/4, λ/2, λ/8 Lines - Impedance Transformations, Smith Chart – Configuration and Applications, Single Stub Matching.

<u>TEXT BOOKS:</u>

- Engineering Electromagnetics – William H. Hayt Jr. and John A. Buck, 8th Ed., McGrawHill,2014
- Principles of Electromagnetics – Matthew N.O. sadiku and S.V. Kulkarni, 6th Ed., Oxford University Press, Aisan Edition, 2015.

<u>.REFERENCES:</u>

- Electromagnetic Waves and Radiating Systems – E.C. Jordan and K.G. Balmain, 2nd Ed. 2000, PHI
- Engineering Electromagnetics – Nathan Ida, 2nd Ed., 2005, Springer (India) Pvt. Ltd., New Delhi
- Transmission Lines and Networks – Umesh Sinha, Satya Prakashan, 2001,(Tech. IndiaPublications), New Delhi.

<u>GATE SYLLABUS:</u>

Maxwell's equations: differential and integral forms and their interpretation, boundary conditions, wave equation, Poynting vector. Plane waves and properties: reflection and refraction, polarization, phase and group velocity, propagation through various media, skin depth, Expressions for Characteristics Impedance , Propagation Constant, Phase and Group Velocities Losslessness/Low Loss Characterization, Distortion - Condition for Distortionlessness and Minimum Attenuation, Input Impedance Relations, SC and OC Lines, Reflection Coefficient, VSWR. UHF Lines as Circuits Elements; λ/4, λ/2, λ/8 Lines

IES SYLLABUS:

Electronics and Tele Communications Engineering: Electro magnetic

Elements of vector calculus, Maxwell's equations-basic concepts; Gauss', Stokes' theorems; Wave propagation through different media, Waveguides-basics, rectangular types, modes, cut-off frequency, dispersion, dielectric types, SC and OC Lines, Reflection Coefficient, VSWR. UHF Lines as Circuits Elements; $\lambda/4$, $\lambda/2$, $\lambda/8$ Lines

(5) SUJECT EXPERTS DETAILS:

INTERNATIONAL:

Arokiaswami Alphones

Associate Professor

School of Electrical and Electronic Engineering

Nanyang Technological university

Singapore

E-mail – ealphones@ntu.edu.sg

Phone – (+65) 67904486, (+91) 9500195198.

NATIONAL :

Dr. B. Manimegalai

Professor

Department of ECE

Thiagarajar College of Engineering

Madurai, Tamil Nadu

Email - naveenmegaa@tce.edu

Mobile - +91-9865191244

Dr. D. Sriram Kumar
Professor
Department of Electronics and Communication Engineering
National Institute of Technology Tiruchirappalli

India - 620015

E- mail – srk@nitt.edu

Mobile - +91-

Dr. T. Shanmuganantham

Associate Professor

School of engineering and technology

Pondicherry University

E- mail - shanmuga.dee@pondiuni.edu.in

Mobile +91 9486640168

INDUSTRY:

Dr. C. Gokulnath

Senior Research RF Engineer

HCL Technologies

Chennai

E-mail – gokul.kesav@gmail.com

Mobile - +91-9943092095

(6) JOURNAL WITH MIN 5 REF PAPERS FOR LITERATURE SURVEY STUDY:

1. **Journal of Electromagnetic waves and applications - https://www.tandfonline.com/toc/tewa20/current**
2. **Electromagnetics https://www.tandfonline.com/toc/uemg20/current**
3. IEEE Transactions on Electromagnetic Compatibility
4. IEEE Transactions on Microwave Theory and Techniques
5. IEEE Transactions on Antennas and Propagation

(7) Lesson Plan:

S.NO	Topic	No. of Periods	Text Books Referred	Method of Teaching
Unit I Electrostatics				
1	Introduction, coordinate systems	1	T1	M1, M2
2	Vector calculus	1	T1,R2	M1, M2
3	Differential vector component	1	T1,R1	M1, M2
4	Lapalcian, poission's euations	1	T1,R2	M1, M2
5	Coulomb's Law	1	T1,R1	M1, M2
6	Electric Field Intensity – Fields due to Different Charge Distributions	1	T1,R1	M1, M2
7	Electric Flux Density, Gauss Law and Applications	2	T1,R1	M1, M2
8	Electric Potential,	1	T1,R1	M1, M2
9	Relations Between E and V, Maxwell's Two Equations for Electrostatic Fields	1	T2,R1	M1, M2
10	Energy Density	1	T1,R2	M1, M2
11	Convection and Conduction Currents	1	T1,R2	M1, M2

12	Dielectric Constant, Isotropic and Homogeneous Dielectrics	**1**	**T1,R1**	**M1, M2**
13	Continuity Equation, Relaxation Time	**1**	**T1,R1**	**M1, M2**
14	Poisson's and Laplace's Equations; Capacitance – Parallel Plate	**1**	**T1,R1**	**M1, M2**
15	Coaxial, Spherical Capacitors	**2**	**T1,R1**	**M1, M2**
Total no of classes:17				
UNIT II: Magneto statics				
18	Ampere's Circuital Law and Applications	**3**	**T1**	**M1, M2**
19	Magnetic Flux Density, Maxwell's Two Equations for Magnetostatic Fields	**1**	**T2,R1**	**M1, M2**
20	Magnetic Scalar and Vector Potentials	**1**	**T1,R1**	**M1, M2**
21	Forces due to Magnetic Fields, Ampere's Force Law	**2**	**T2**	**M1, M2**
22	Faraday's Law and Transformer EMF	**1**	**T1**	**M1, M2**
23	Inconsistency of Ampere's Law and Displacement Current Density	**2**	**T1,T2,R1**	**M1, M2**
Total no of classes:11				
UNIT-III: Maxwell's Equations(Time Varying Fields)				
24	Maxwell's Equations in Different Final Forms and Word Statements	**4**	**T1,R1**	**M1, M2**
25	Conditions at a Boundary Surface : Dielectric- Dielectric	**4**	**T1,R1**	**M1, M2**
26	Dielectric-Conductor Interfaces,	**3**	**T1,R1**	**M1, M2**
Total no of classes:11				
UNIT-IV:EM Wave Characteristics				

27	Wave Equations for Conducting and Perfect Dielectric Media	**1**	**T2,R1**	**M1, M2**
28	Uniform Plane Waves – Definition, All Relations Between E & H	**2**	**T1**	**M1, M2**
29	Sinusoidal Variations	**1**	**T2**	**M1, M2**
30	Wave Propagation in Lossless and Conducting Media	**2**	**T2,R1**	**M1, M2**
31	Conductors & Dielectrics	**1**	**T2,R1**	**M1, M2**
32	Wave Propagation in Good Conductors and Good Dielectrics	**1**	**T1,R1**	**M1, M2**
33	Polarization	**1**	**T1,R1**	**M1, M2**
34	Reflection and Refraction of Plane Waves – Normal and Oblique Incidences for both Perfect Conductor and Perfect Dielectrics	**2**	**T2,R1**	**M1, M2**
35	Brewster Angle, Critical Angle and Total Internal Reflection	**1**	**T1,R1**	**M1, M2**
36	Surface Impedance, Poynting Vector and Poynting Theorem – Applications	**3**	**T2,R1**	**M1, M2**
			Total no of classes:15	
		UNIT-V:Transmission Lines		
38	Transmission Line Equations	**2**	**T2,R2**	**M1, M2**
39	Conditions for Distortion andDistortion less transmission lines	**3**	**T1,R2**	**M1, M2**
40	Types of Loading	**2**	**T1,R2**	**M1, M2**
41	SC,OC and λ/4, λ/2, λ/8 Lines	**3**	**T1,R2**	**M1, M2**
42	Smith chart	**3**	**T1,R2**	**M1, M2**
43	Stub Matching	**2**	**T2,R2**	**M1, M2**
			Total no of classes:15	

Teaching methods:

M1 : Black Board

M2 : ICT Methods (PPT /E-Resources / NPTEL)

(8) SUGGESTED BOOKS:

TEXT BOOKS:

- Engineering Electromagnetics – William H. Hayt Jr. and John A. Buck, 8th Ed., McGrawHill,2014
- Principles of Electromagnetics – Matthew N.O. sadiku and S.V. Kulkarni, 6th Ed., Oxford University Press, Aisan Edition, 2015.

REFERENCES:

- Electromagnetic Waves and Radiating Systems – E.C. Jordan and K.G. Balmain, 2nd Ed. 2000, PHI
- Engineering Electromagnetics – Nathan Ida, 2nd Ed., 2005, Springer (India) Pvt. Ltd., New Delhi

(9) WEBSITES FOR SELF LEARNING RESOURCES:

1. NPTEL VIDEO LECTURES: https://nptel.ac.in/courses/108/106/108106073/

2. COURSERA:

https://www.coursera.org/lecture/electrodynamics-introduction/1-1-introduction-to-electromagnetism-qiIQb

3. MIT OPEN COURSEWARE:

https://ocw.mit.edu/courses/physics/8-311-electromagnetic-theory-spring-2004/index.htm

(10) QUESTION BANK:

UNIT 1 – ELECTROSTATICS

Short answers questions

1. State stokes theorem.

The line integral of a vector around a closed path is equal to the surface integral of the normal component of its curl over any surface bounded by the path

$$\int H.dl = \int\int (\Delta x H) ds$$

Where, H=Magnetic field intensity

2. State the condition for the vector F to be solenoidal.

Del .F =0

Where, F=A i + B i + C i

3. State the condition for the vector F to be irrotational.

Del x F =0

Where, F=A i + B i + C i

4. Give the relationship between potential gradiant and electric field.

$$E = -\,\mathrm{del}V$$

Where, E=Electric Field Intensity

V=Electric Potential

5. What is the physical significance of div D ?

The divergence of a vector flux density is electric flux per unit volume leaving a small volume. This is equal to the volume charge density.

6. What are the sources of electric field and magnetic field?

Stationary charges produce electric field that are constant in time, hence the term electrostatics. Moving charges produce magnetic fields hence the term magneto statics.

7. State Divergence Theorem.

The integral of the divergence of a vector over a volume v is equal to the surface integral of the normal component of the vector over the surface bounded by the volume.

$$\iint \mathrm{Del}.F\, ds = \iiint F\, dv$$

Where, F=A i + B i + C i

8. Define divergence.

The divergence of a vector F at any point is defined as the limit of its surface integral per unit volume as the volume enclosed by the surface around the point shrinks to zero.

9. State coulombs law.

Coulombs law states that the force (F) between any two point charges (Q1 and Q2) is directly proportional to the product of their magnitudes and inversely proportional to the square of the distance ® between them. It is directed along the line joining the two charges.

$$F= (Q1Q2) / (4\Pi\varepsilon r^2) \, dr$$

10. State Gauss law for electric fields

The total electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

11. Define electric flux.

The lines of electric force is known as electric flux.

12. Define electric flux density.

Electric flux density is defined as electric flux per unit area.

$$\text{Electric flux density}=\phi/A$$

13. Define electric field intensity.

Electric field intensity is defined as the electric force per unit positive charge. $E =F/ Q$

$=Q/4\Pi\varepsilon r^2$ V/m

14. Name few applications of Gauss law in electrostatics.

Gauss law is applied to find the electric field intensity from a closed surface. Ex:

Electric field can be determined for shell, two concentric shell or cylinders etc.

15. What is electrostatic force?

The force between any two particles due to existing charges is known as electrostatic force, repulsive for like and attractive for unlike.

16. What are dielectrics?

Dielectrics are materials that may not conduct electricity through it but on applying electric field induced charges are produced on its faces .The valence electron in atoms of a dielectric are tightly bound to their nucleus.

17. What is a capacitor?

A capacitor is an electrical device composed of two conductors which are separated through a dielectric medium and which can store equal and opposite charges ,independent of whether other conductors in the system are charged or not.

18. Define dielectric strength.

The dielectric strength of a dielectric is defined as the maximum value of electric field that can b applied to the dielectric without its electric breakdown.

19. What meaning would you give to the capacitance of a single conductor?

A single conductor also possess capacitance. It is a capacitor whose one plate is at infinity.

20. Why water has much greater dielectric constant than mica.?

Water has a much greater dielectric constant than mica .because water has a permanent dipole moment, while mica does not have.

21. What is a point charge?

Point charge is one whose maximum dimension is very small in comparison with any other length.

22. Define linear charge density(ρ1).

It is defined as the charge (Q) per unit length (L).

$$\rho 1 = Q/L$$

23. Define potential difference.

Potential difference is defined as the work done in moving a unit positive charge from one point to another point in an electric field.

24. Define potential.

Potential at any point is defined as the work done in moving a unit positive charge from infinity to that point in an electric field.

$$V = Q / 4\Pi\varepsilon r^2$$

Where, V=Electric Potential

Q=Charge

ε= Relative permittivity

r=Distance between charge

1. **Give the relation between electric field intensity (E) and electric flux density (D).** $D = \varepsilon E$ C/m2

2. **Write the expression for energy density in electrostatic field.**

$$W = 1 / 2\ \varepsilon E^2$$

Where, V=Electric Potential

W=Energy Density

3. **Write the boundary conditions at the interface between two perfect dielectrics.**

a. The tangential component of electric field is continuous i.e) $E_{t1}=E_{t2}$
b. The normal component of electric flux density is continuous i.e) $D_{n1}=D_{n2}$

4. Write down the expression for capacitance between two parallel plates.

$$C=\varepsilon A / d$$

Where, C=Capacitance

A=Area

d= Distance between charge

5. State amperes circuital law.

Magnetic field intensity around a closed path is equal to the current enclosed by the path.

$$H.dl=I$$

Where, H=Magnetic field intensity

I= Current

Long answers questions

1. Given that potential $V=10\sin\theta\cos\Phi/r2$ find the electric flux density D at $(2,\pi/2,0)$

2. Derive an expression for the electric field due to a straight and infinite uniformly charged wire of length 'L' meters and with a charge density of $+\lambda$ c/m at a point P which lies along the perpendicular bisector of wire.

3. Explain poissons and lapace's equations.

4. A uniform line charge ρL =25Nc/m lies on the x=3m and y=4m in free space .Find the electric field intensity at a point (2,3,15)m.

5. Obtain the expression for the energy stored in a capacitor.

6. Drive an expression for energy stored and energy density in an electrostatic field.

7. Derive an expression for the capacitance of two wire transmission line.

8 Derive an expression for capacitance of concentric spheres.

9. Derive an expression for capacitance of co-axial cable.

10 .Explain and derive the polarization of a dielectric materials.

11. List out the properties of dielectric materials.

12. Derive an expression for series and parallel plate capacitor.
13. The electric field in a spherical co-ordinate is given by E=rρr/5ε. Show that closed ∫E.dS=∫(▼.E)Dv.
14. State and proof divergence theorem and strokes theorem.
15. Check validity of the divergence theorem considering the field D=2xy ax +x2ay c/m2 and the rectangular parallelepiped formed by the planes x=0,x=1,y=0,y=2 & z=0,z=3.
16. A vector field D=[5r2/4]Ir is given in spherical co-ordinates. Evaluate both sides of divergence theorem for the volume enclosed between r=1&r=2.
17. Given A= 2r cosΦ+Riφ in cylindrical co-ordinates for the contour x=0to1 y=0to1 , verify stoke's theorem.
18. Explain three co-ordinate system.
19. State and proof gauss law .and explain applications of gauss law.

UNIT 2 –MAGNETOSTATICS

Short answers questions

1.State Biot –Savarts law.

It states that the magnetic flux density at any point due to current element is proportional to the current element and sine of the angle between the elemental length and inversely proportional to the square of the distance between them

$$dB=\mu 0 \text{ Idl } \sin\theta / 4\Pi \varepsilon r^2$$

Where, B=Magnetic field density

I= Current

r= Distance between charge

ε= Relative permittivity

2. What are the significant physical differences between Poisson 's and laplace's equations.

Poisson's and laplace's equations are useful for determining the electrostatic potential V in regions whose boundaries are known.

When the region of interest contains charges poissons equation can be used to find the potential.

3. Give the expression for electric field intensity due to a single shell of charge E = Q / 4Πεr^2

Where, E=Electric field indensity

r= Distance between charge

ε= Relative permittivity

4. Give the expression for potential between two spherical shells

V= 1/ 4Π (Q1/a – Q2/b)

5. Define electric dipole.

Electric dipole is nothing but two equal and opposite point charges separated by a finite distance.

6. How is electric energy stored in a capacitor?

In a capacitor, the work done in charging a capacitor is stored in the form of electric energy.

7. Define current density.

Current density is defined as the current per unit area.

J= I/A Amp/m2

8. What is meant by displacement current?

Displacement current is nothing but the current flowing through capacitor. J= D / t

9. State point form of ohms law.

Point form of ohms law states that the field strength within a conductor is proportional to the current density.

$$J = \sigma E$$

10. Define surface charge density.

It is defined as the charge per surface area.

$$\rho S = Q/S$$

11. Define magnetic vector potential.

It is defined as that quantity whose curl gives the magnetic flux density.

12. Write down the expression for magnetic field at the centre of the circular coil.

$$H = I/2A.$$

Where, H=Magnetic Field Intensity

I=Current

A=Area

13. Give the relation between magnetic flux density and magnetic field intensity.

$$B = \mu H$$

Where, H=Magnetic Field Intensity

B=Magnetic Field Density

14. Write down the magnetic boundary conditions.

i) The normal components of flux density B is continuous across the boundary.

ii) The tangential component of field intensity is continuous across the boundary.

15. Give the force(F) on a current element (dl).

$$dF = BIdl\sin\theta$$

Where, B=Magnetic Field Density

I=Current

16. Define magnetic moment.

Magnetic moment(m) is defined as the maximum torque (T) per magnetic induction of flux density (B).

$$m=T/B$$

17. State Gauss law for magnetic field.

The total magnetic flux passing through any closed surface is equal to zero.

$$B.ds = 0$$

Where, B=Magnetic Field Density

18. Define magnetic field strength.

The magnetic field strength (H) is a vector having the same direction as magnetic flux density (B).

$$H=B/\mu$$

19. Give the formula to find the force between two parallel current carrying conductors.

$$F=\mu I1.I2 / 2\Pi R$$

Where, F=Force   I=Current

R=Distance between charge

20. Give the expression for torque experienced by a current carrying loop situated in a magnetic field.

$$T = IAB\sin\theta$$

Where, T=Torque

I=Current

A=Area

B=Magnetic Field Density

21. What is lorentz force?

Lorentz force is the force experienced by the test charge .It is maximum if the direction of movement of charge is perpendicular to the orientation of field lines.

22.Explain the conservative property of electric field.

The work done in moving a point charge around a closed path in a electric field is zero. Such a field is said to be conservative.

$$E.dl = 0$$

23. Write the expression for field intensity due to a toroid carrying a filamentary current I.

$$H=NI / 2\Pi R$$

Where,

H= Magnetic Field Intensity

N=Number of Turns

I=Current

R=Distance between charges

24. What are equipotential surfaces?

An equipotential surface is a surface in which the potential energy at every point is of the same vale.

19. Define loss tangent.

Loss tangent is the ratio of the magnitude of conduction current density to displacement current density of the medium.

20. Define reflection coefficients.

Reflection coefficient is defined as the ratio of the magnitude of the reflected field to that of the incident field.

21. Define transmission coefficients.

Transmission coefficient is defined as the ratio of the magnitude of the transmitted field to that of incident field.

6. What will happen when the wave is incident obliquely over dielectric – dielectric boundary?

When a plane wave is incident obliquely on the surface of a perfect dielectric part of the energy is transmitted and part of it is reflected .But in this case the transmitted wave will be refracted, that is the direction of propagation is altered.

7. What is the expression for energy stored in a magnetic field?

$$W = \frac{1}{2} LI2$$

24. What is energy density in

magnetic field? $W = \frac{1}{2} \mu H2$

25. Distinguish between solenoid and toroid.

Solenoid is a cylindrically shaped coil consisting of a large number of closely spaced turns of insulated wire wound usually on a non magnetic frame. If a long slender solenoid is bent into the form of a ring and there by closed on itself it becomes a toroid.

Long answers questions

1. Derive the expressions for magnetic field intensity due to finite and infinite line.
2. Derive the expressions for magnetic flux intensity due to solenoid of the coil.
3. Derive the expressions for magnetic field intensity due to toroidal coil and circular coil.
4. Derive an expression for energy stored and energy density in magnetic field.
5. Derive an expression for self inductance of two wire transmission line.
6. Derive an expression for force between two current carrying conductors.
7. Derive an expression for co-efficient of coupling.
8. Explain Magnetic materials and scalar and vector magnetic potentials.
9. Derive the expressions for boundary conditions in magnetic fields.
10. Derive the expression for torque developed in a rectangular closed circuit carrying current I a uniform field.

UNIT 3 – MAXWELL'S EQUATIONS (TIME VARYING FIELDS)

Short answers questions

1. State Maxwell's fourth equation.

The net magnetic flux emerging through any closed surface is zero.

2. State Maxwell's Third equation

The total electric displacement through the surface enclosing a volume is equal to the total charge within the volume.

3. State the principle of superposition of fields.

The total electric field at a point is the algebraic sum of the individual electric field at that point.

4. Define ohms law at a point

Ohms law at appoint states that the field strength within a conductor is proportional to current density.

5. Define self inductance.

Self inductance is defined as the rate of total magnetic flux linkage to the current through the coil.

6. Define pointing vector.

The vector product of electric field intensity and magnetic field intensity at a point is a measure of the rate of energy flow per unit area at that point.

7. State Lenz law.

Lenz’s law states that the induced emf in a circuit produces a current which opposes the change in magnetic flux producing it.

8. What is the effect of permittivity on the force between two charges?

Increase in permittivity of the medium tends to decrease the force between two charges and decrease in permittivity of the medium tends to increase the force between two charges.

9. State electric displacement.

The electric flux or electric displacement through a closed surface is equal to the charge enclosed by the surface.

10. What is displacement flux density?

The electric displacement per unit area is known as electric displacement density or electric flux density.

11. What is the significance of displacement current?

The concept of displacement current was introduced to justify the production of magnetic field in empty space. It signifies that a changing electric field induces a magnetic field .In empty space the conduction current is zero and the magnetic fields are entirely due to displacement current.

12. Distinguish between conduction and displacement currents.

The current through a resistive element is termed as conduction current whereas the current through a capacitive element is termed as displacement current.

13. Define inductance.

The inductance of a conductor is defined as the ratio of the linking magnetic flux to the current producing the flux.

$$L = N\Phi / I$$

14. What is main cause of eddy current?

The main cause of eddy current is that it produces ohmic power loss and causes local heating.

15. How can the eddy current losses be eliminated?

The eddy current losses can be eliminated by providing laminations. It can be proved that the total eddy current power loss decreases as the number of laminations increases.

16.What is the fundamental difference between static electric and magnetic field lines?

There is a fundamental difference between static electric and magnetic field lines .The tubes of electric flux originate and terminates on charges, whereas magnetic flux tubes are continuous.

Long answers questions

1. Explain the relation between field theory and circuit theory.
2. Derive an expression for displacement, conduction current densities. Also obtain an expression for continuity current relations
3. Derive all the maxwells equations. i)Maxwells equation from electric Gauss law.
    ii) Maxwells equation from magnetic Gauss law. iii)Maxwells equation from Amperes law.

iv) Maxwells equation from Faradays law.

4. State and explain Faradays and Lenzs law of induction and derive maxwells equation.

UNIT IV- ELECTROMAGNETIC WAVE CHARACTERISTICS

Short answers questions

1. Define a wave.

If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times , the time delay being proportional to the space separation from the first location then the group of phenomena constitutes a wave.

2. Mention the properties of uniform plane wave.

i) At every point in space ,the electric field E and magnetic field H are perpendicular to each other.

ii)The fields vary harmonically with time and at the same frequency everywhere in space.

3. Define intrinsic impedance or characteristic impedance.

It is the ratio of electric field to magnetic field.or It is the ratio of square root of permeability to permittivity of medium.

4. Give the characteristic impedance of freespace.

377ohms

5. Define skin depth

It is defined as that depth in which the wave has been attenuated to 1/e or approximately 37% of its original value.

6. Define Poynting vector.

The pointing vector is defined as rate of flow of energy of a wave as it propagates.

$$P = E \times H$$

7. State Poynting Theorem.

The net power flowing out of a given volume is equal to the time rate of decrease of the the energy stored within the volume- conduction losses.

8. Give significant physical difference between poisons and laplace's equations.

When the region contains charges poisons equation is used and when there is no charges laplaces equation is applied.

9. Give the difficulties in FDM.

FDM is difficult to apply for problems involving irregular boundaries and non homogenious material properties.

10. Explain the steps in finite element method.

i) Discretisation of the solution region into elements. ii) Generation of equations for fields at each element iii) Assembly of all elements

iv) Solution of the resulting system

11. What are uniform plane waves?

Electromagnetic waves which consist of electric and magnetic fields that are perpendicular to each other and to the direction of propagation and are uniform in plane perpendicular to the direction of propagation are known as uniform plane waves.

12. Write short notes on imperfect dielectrics.

A material is classified as an imperfect dielectrics for ó <<ùå, that is conduction current density is small in magnitude compared to the displacement current density.

13.What is the significant feature of wave propagation in an imperfect dielectric ?

The only significant feature of wave propagation in an imperfect dielectric compared to that in a perfect dielectric is the attenuation undergone by the wave.

14.What is the major drawback of finite difference method?

The major drawback of finite difference method is its inability to handle curved

boundaries accurately.

h and bound algorithm?

Long answers questions

1. A uniform plane wave of 200 MHz, traveling in free space Impinges normally on a large block of material having εr =4 , μr =9 and σ =0. Calculate transmission and reflection co efficient of interface.

2.What are the different ways of EMF generation? Explain with the governing equations and suitable practical examples.

3. With necessary explanation, derive the maxwell’s equation in differential and integral forms

4. Write short notes on faradays law of electromagnetic induction.

5. What do you mean by displacement current? write down the expression for the total current density

6.In a material for which $\sigma=5$ s/m and $\varepsilon r=1$ and E=250 sin 1010t (V/m). find the conduction and displacement current densities.

7.Find the total current in a circular conductor of radius 4mm if the current density Varies according to J=104/R A/m2 .

8. The magnetic field intensity in free space is given as H=H0sinθ ay t A/m. where θ=ωt-βz and β is a constant quantity. Determine the displacement current density.

9. Show that the ratio of the amplitudes of the conduction current density and displacement current density is σ/ωε,for the applied field amplitude ratio if the applied field is E=Em e-t/λ where λ is real.

10. Calculate the attenuation constant and phase constant for the uniform plane wave with the frequency of 10GHz in a medium for which μ=μ0, εr=2.3 and σ=2.54x10^-4Ω/m

11. Derive the expression for the attenuation constant, phase constant and intrinsic impedance for a uniform plane wave in a good conductor.

12. Derive the one dimensional general wave equation and find the solution for wave equation.

13. Discuss about the plane waves in lossy dielectrics.

14. Discuss about the plane waves in lossless dielectrics.
15. Briefly explain about the wave incident

(i) Normally on perfect conductor

(ii) Obliquely to the surface of perfect conductor.

16. Briefly explain about the wave incident

(i) Normally on perfect dielectrics

(ii) Obliquely to the surface of perfect dielectrics.

17. Assume that Eland H waves, travelling in free space, are normally Incident on the interface with a perfect dielectric with εr=3 .calculate the magnitudes of incident, reflected and transmitted E and H waves at the interface.

Unit V Transmission Lines

(11) CASE STUDY

Project 1:-

TITLE:- Effects of Electromagnetic Radiation on Biological Systems: A short review of case studies

Abstract:-

The use of electromagnetic radiations (EM) has been increased many fold in the recent times mainly due to the advances in information technology (wireless and satellite communication). The enhanced presence of these EM radiations in space is causing concerns of EM interference not only of several equipment hut also of biological activity. There is a serious concern on the effect of the EM radiation on biological systems in general and on human systems in particular, specifically when the exposure levels are rather high. There are guidelines, arrived after experimentation, for the minimum exposure levels of these EM radiations for safe working. The problem is twofold first, to measure and standardize the radiation levels and second, to evaluate the effects on biological systems. The latter part is more difficult. There are several case studies conducted on the effect of non-ionizing radiations on biological systems over a wide range of frequencies emitted either from antennas, video displays or even from transmission lines. It is also recorded that the radiations from plasma seems to have desirable effects on curing Cancer. Present paper summarizes three case studies conducted recently. Though conclusive evidence is yet to he arrived, it is observed that in most of the cases, the non-ionizing electromagnetic radiation seems to affect the biological activity; the experiments have shown that in the case of osteoporosis the EM radiations are observed to have a curing effect.

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Project 2:-

TITLE:- Electromagnetic Compatibility: Analysis and Case Studies in Transportation

Abstract:-

This case study is a mathematically-rich extension of courses required to maintain the Federal Communications Commission (FCC), the Canadian Standards Association (CSA), and the European Union certifications. The text provides an in-depth study of the electromagnetic compatibility (EMC) issues related to specific topics in transportation and communications, including Light Rail Transit, shadow effects, and radio dead spots, through the analysis of real-world case studies in the United States and Europe. It provides Cartesian, cylindrical, and spherical solutions that can be applied to Maxwell's and Wave Equations. This case study covers topics such as SCADA Systems, shielding, and complexities of radio frequencies and their effect on communication houses and also provides information for alternative industries to apply the solutions from the case studies and background content to their own professions.

12) Mid –I Assignment

Set-I

1. Point charges 5nC and -2nC are located at (2, 0, 4) and (-3, 0, 5) respectively. Determine the force on a 1nc point charge located at (1, -3, 7).

2. Find the electric field intensity and electric flux density due to an infinite surface charge of

charge density ρ s C/m 2

3. Obtain the expression for H at any point due to an infinite sheet of current density K A/m

using Ampere's law

4. Define Magnetic flux density and obtain Maxwell's Fourth equation

5. a) Distinguish between electric fields and Magnetic fields

b)Two wires carrying currents in same direction of 5000A and 10000 A are placed with 5cm

apart distance. Calculate force between them in Newton per unit length

6.Explain about a) Continuity equation b) Relaxation time

Set-II

1. a) State coulombs law and Derive expressions for force and electric field intensity due to two charges and multiple charges

b) Obtain expression for the electric field intensity due to volume charge of charge density

ρ v C/m 3

2. Obtain the expression for H at any point due to an infinitely line coaxial transmission line

using Ampere's law

3. Planes Z = 0 and Z = 4 carry current K = -l0ax A/m and K = l0ax A/m, respectively.

Determine H at a) (1 ,1 ,1) b) (0 ,-3, 10)

4. a) State and prove Ampere's circuital law.

b) Explain the Biot-savart's law in detail.

5. State Gauss law and Find the electric field intensity due to an infinite line charge whose charge density is ρ L C/m using Gauss law .

Set -3

1. Derive the expression for capacitance of a spherical capacitor

2. Obtain expression for the electric field intensity due to infinite line charge of charge density

ρ L C/m

3.a) Obtain the expression for H due to infinite long straight conductor at a point P ‘r ‘meters

away from the conductor using ampere circuital law.

4. Derive the expression for the force between two current elements.

5. Define and explain the terms scalar and vector magnetic potential. How to determine these

quantities for a magnetic field.

6. State Gauss law and Find the electric field intensity due to an infinite surface charge of charge density ρ s C/m 2 using Gauss law .

Mid-II Assignment

Set -I

1) Define Faraday’s Laws? What are the different ways of emf generation ? Also derive

differential and integral form of Faraday’s Laws

2) Derive the magnetic boundary condition at the interface between two perfect dielectrics

3) Determine the general solution for Uniform plane wave equation

4)Explain lossless/Distortion less lines and also derive the input impedance for transmission lines

5) What is loading? Explain Different types of loading.

Set -II

1) Derive electric boundary condition between two perfect dielectric medium?

2) Derive expression for Uniform plane wave equation

3) Derive the relationship between E and H in Uniform plane wave?

4) Explain and derive the expression for R and X circle equations of Smith chart and its applications.

5a) Explain reflection and derive reflection coefficient and also derive the relation between reflection coefficient and VSWR.

Set –III

1) a) Derive electric boundary condition between conductor and dielectric interface?

b) Obtain the condition at the boundary conductor and free space interface?

2) Write Maxwell's equations in both differential and integral form for time varying fields?

3) State and prove Pointing theorem

4) Derive the Wave equation for conducting medium?

5) a) Explain the construction of Microstirp line and working?

b) Obtain the expression for characteristic impedance and effective dielectric constant of a

microstrip line

9. Sample assignment script

13) List of topics for student's seminars:

- Gauss law and Divergence Theorem
- Capacitors and Types of capacitors
- Dielectric Materials
- Biot-savart’s Law
- Ampere Circuit law
- Transformer EMF
- Normal and Oblique Incidence
- Brewster Angle and Snell’s law

(14) STEP/COURSE MATERIAL:

https://drive.google.com/drive/folders/15IH-kw77IrF7amjaRkyDga5Bqojhp0tQ?usp=sharing

(15) EXPERT LECTURE WITH TOPICS & SCHEDULES

S.NO	SUBJECT	TOPIC	YEAR	RESOURCE PERSON	DATE
1	EMTL	EM Theory	II-II	Others	10/032024 (TENTATIVE)
2	EMTL	Transmission Lines	II-II	Others	06/04/2024 (TENTATIVE)

LINEAR AND DIGITAL IC APPLICATIONS

Subject Code: EC403PC

Class: II Year B.Tech ECE II Semester

BY

G.KALPANA
ASSISTANT PROFESSOR

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

CMR ENGINEERING COLLEGE

S.NO		**CONTENT**
(1)	**-**	**Preamble/Introduction**
(2)	**-**	**Prerequisites**
(3)	**-**	**Objectives and Outcomes**
(4)	**-**	**Syllabus** **1.JNTU/R22-CMREC** **2. GATE** **3. IES**
(5)	**-**	**List of Expert Details** (Local/National/International with Contact details/Profile link/Blogs/their research Contribution towards the subject)
(6)	**-**	**Journals with min 5 ref paper for literature study**
(7)	**-**	**Subject -Lesson plan**
(8)	**-**	**Suggested Books (**prescribed and References)
(9)	**-**	**Websites for self learning Resources like** *www.geeksforgeeks.org, www.schools.com, Coursera ,edX, Udemy, Khan Academy*, NPTEL etc along Registration procedures*)*
(10)	**-**	**Question Banks** **1.JNTUH/Model papers** **2.GATE**
(11)	**-**	**Two case study presentations with Project / Product/ Model /prototypes/ Industrial applications.**
(12)	**-**	**Assignment Question/Innovative Assignments sets.**
(13)	**-**	**List of topics for students Seminars with Guidelines**
(14)	**-**	**STEP/Course material in softcopy**
(15)	**-**	**Expert Lectures with topics & Schedules(if any)**

1. **Preamble/Introduction:**

Learners would find that on completion of these course they would be able design more complicated circuits, as a large number of circuits are readily available as monolithic ICs and that they would be able to make the right choice of ICs by referring the data sheets of these ICs which are readily available over the net..

Linear and Digital IC Applications is being applied in many fields in today's world,

- Radar
- Wristwatches
- Computers
- Video processor
- Televisions
- Logic devices
- Juice makers
- Memory devices
- Audio amplifiers
- Microwave amplifiers
- Small signal amplifiers
- Radiofrequency decoders and encoders
- Voltage regulators
- Timers
- Clock chips
- Calculators
- Flip flops
- Memory chips
- Counters
- Temperature sensors
- Microcontrollers

2. Prerequisites:

This subject recommends basic knowledge & practice on

This subject recommends continuous practice of various simple arithmetic operations like addition, subtraction. It needs requisite knowledge about assuming and designing the logic circuits. How to reduce the size and complexity of the digital circuits.

Course Objectives: The main objectives of the course are:

1. To introduce the basic building blocks of linear integrated circuits.
2. To introduce the theory and applications of Analog multipliers and PLL.
3. To introduce the concept sine waveform generation and introduce some special

function ICs.
4. To understand and implement the working of basic digital circuits.

Course Outcomes: Upon completing this course, the students will be able to

1. A thorough understanding of operational amplifiers with linear integrated circuits.
2. Attain the knowledge of functional diagrams and design applications of IC555 and IC565.
3. Acquire the knowledge and design the Data converters.
4. Choose the proper digital integrated circuits by knowing their characteristics.

4. SYLLABUS – JNTU-R22/UGC AUTONOMOUS

UNIT-I

Operational Amplifier: Ideal and practical Op-amp, OP-Amp characteristics, DC and AC characteristics, Features of 741 Op-amp, Modes of Operation-Inverting, Non-inverting, Differential, Instrumentation amplifier, AC amplifier, Differentiators and Integrators, Comparators, Schmitt Trigger, Introduction to voltage regulators, Features of 723 Regulator, Three Terminal Voltage Regulators.

UNIT – II

OP- AMP, IC-555 & IC-565 Applications: Introduction to Active Filters, Characteristics of Band pass, Band reject and all pass filters, Analysis of 1st order LPF & HPF Butterworth Filters, Waveform Generators- Triangular, Sawtooth, Square wave, IC555 Timer- functional diagram, Monostable and Astable operations, Applications, IC565 PLL- Block Schematic, Description of individual blocks, Applications

UNIT – III

Data Converters: Introduction, Basic DAC techniques, Different types of DACs-weighted resistor DAC, R-2R ladder DAC, inverted R-2R DAC, Different types of ADCs - parallel comparator type ADC, counter type ADC, successive approximation ADC and dual slope ADC, DAC and ADC Specifications

UNIT – IV

Combinational logic ICS: Specifications and Applications of TTL-74XX & CMOS 40XX Series ICs- Code Converters, Decoders, Demultiplexers, LED & LCD Decoders with Drivers, Encoders, Priority Encoders, multiplexers, Priority Generators/Checkers, Parallel Binary Adder/Subtractor, Magnitude Comparators.

UNIT – V

Sequential Logic IC's and Memories: Familiarity with commonly available 74XX & CMOS 40XX Series ICs- All types of Flip-Flops, Synchronous Counters, Decade Counters, Shift Registers.
Memories-ROM Architecture, Types of ROMS & Applications, RAM Architecture, Static & Dynamic RAMs.

5.List of Expert Details:

The Expert Details which have been mentioned below are only a few of the eminent ones known Internationally, Nationally and Locally. There are a few others known as well.

INTERNATIONAL

1.Prof.K **Subbarangaiah** is Director of VEDA IIT, Hyderabad

NATIONAL

1. Dr.K. LAL KISHORE, Ph.D., MIEEE, FIETE, MISTE, MISHM, JNTU, Hyderabad
2. Mr .Sundaram, AGM, CAD R&D, ECIL, Hyderabad..
3. Mr. Rajendra naik, Asst Prof, Dept of ECE, Osmania University, Hyderabad.

REGIONAL

1. Dr. N.S.Murthy, Professor and Head Dept. of ECE, REC,Warangal - 506004 (India)
   email: nsm@recw.ernet.in
2. ***S.G Vinayaka Prasad, Sr. App. Engineer, Silicon Micro Systems***
3. DR. M. Madhavi Latha, JNTU, Hyderabad
4. Dr. Sarat Chandra Babu, Centre Head C-DAC, Hyderabad email: Sarat_chandra@hotmail.com

6 JOURNALS

(8) *JOURNALS*

INTERNATIONAL

1. IEEE transactions on DIGITAL DESIGNS

2. IEEE proceedings circuits, devices and systems
3. International journal of circuit designs and applications
4. IEEE transactions on IC design
5. VSI vision

NATIONAL

1. DIGITAL DESGIN MAGAZINE
2. JOURNAL FOR DIGITAL SYSTEMS
3. IBM system magazine

7 SUBJECT (LESSON) PLAN

Topic Name	No. of classes	Text books
UNIT I: OPERATIONAL AMPLIFIER		
Ideal and practical Op-amp	01	T3, T2
OP-Amp characteristics	01	T3, T2
DC and AC characteristics, Features of 741 Op-amp	03	T3, T2
Modes of Operation-Inverting, Non-inverting	02	T3, T2
Differential, Instrumentation amplifier, AC amplifier	02	T3, T2
Differentiators and Integrators	01	
Comparators, Schmitt Trigger	02	T3, T2
Introduction to voltage regulators, Features of 723 Regulator	02	T3, T2
Introduction to voltage regulators, Features of 723 Regulator	01	T3, T2
	15	
UNIT II: OP- AMP, IC-555 & IC-565 APPLICATIONS		
Introduction to Active Filters,	01	T3, T2
Characteristics of Band pass	01	T3, T2
Band reject and all pass filters	02	T3, T2
Analysis of 1st order LPF & HPF Butterworth Filters	03	T3, T2
Waveform Generators- Triangular, Saw tooth, Square wave	03	T3, T2
IC555 Timer- functional diagram	01	T3, R5
Monostable and Astable operations, Applications	02	T3, R5
IC565 PLL- Block Schematic, Description of individual blocks, Applications	02	T3, R5
	15	

UNIT III: DATA CONVERTERS		
Introduction, Basic DAC techniques	02	T3, R5
Different types of DACs-weighted resistor DAC	01	T3, R5
R-2R ladder DAC, inverted R-2R DAC	02	T3, R5
Different types of ADCs - parallel comparator type ADC	01	T3, R5
counter type ADC	01	T3, R5
successive approximation ADC and dual slope ADC	02	T3, R5
DAC and ADC Specifications	01	T3, R5
	10	
UNIT IV: COMBINATIONAL LOGIC ICS		
Specifications and Applications of TTL-74XX & CMOS 40XX Series ICs	02	T3, R5
Code Converters, Decoders, Demultiplexers	03	T3, R5
LED & LCD Decoders with Drivers	01	T3, R5
Encoders, Priority Encoders	02	T3, R5
multiplexers, Priority Generators/Checkers	02	T3, R5
Parallel Binary Adder/Subtractor	02	T3, R5
Magnitude Comparators	01	T3, R5
	13	
UNIT V: SEQUENTIAL LOGIC IC'S AND MEMORIES		
Familiarity with commonly available 74XX & CMOS 40XX Series ICs	01	T3, R5
All types of Flip-Flops	01	T3, R5
Synchronous Counters, Decade Counters	02	T3, R5
Shift Registers	02	T3, R5
Memories-ROM Architecture	01	T3, R5
Types of ROMS & Applications	01	T3, R5
RAM Architecture	01	T3, R5
Static & Dynamic RAMs	02	T3, R5
Total No. of Classes	**11**	
Total No. of Classes	**64**	

8 SUGGESTED BOOKS

TEXT BOOKS:

1. Ramakanth A. Gayakwad - Op-Amps & Linear ICs, PHI, 2003.
2. Floydand Jain- Digital Fundamentals, 8th Ed., PearsonEducation,2005.

REFERENCE BOOKS:

1. D. Roy Chowdhury – Linear Integrated Circuits, New Age International(p)Ltd,2nd Ed., 2003.

2. John. F. Wakerly – Digital Design Principles and Practices, 3rd Ed., Pearson, ,2009.
3. Salivahana -Linear Integrated Circuits and Applications, TMH, 2008.
4. William D.Stanley- Operational Amplifiers with Linear Integrated Circuits, 4th Ed., PearsonEducation India, 2009.

9 WEBSITES

Do not confine yourself to the list of websites mentioned here alone. Be cognizant and keep yourself abreast of the others too. The given list is not exhaustive.

1. www.ieee.org
2. www.2dix.com
3. www.xillinx.com
4. www.cdac.com
5. www.vlsi.edu
6. www.vlsi.iitkgp.ernet.in
7. www.educypedia.be/electronic/digital.com
8. www.iitb.ac.in
9. www.iitm.ac.in
10. www.iitr.ac.in
11. www.iitg.ernet.in
12. www.bits-pilani.ac.in
13. www.metorgraphics.com
14. www.vlsi-reasearch.com
15. www.iisc.ernet.in
16. www.samsung.com
17. www.vedaiit.com

M1 : Lecture Method	**M6 : Tutorial**
M2 : Demo Method	**M7 : Assignment**
M3 : Guest Lecture	**M8 : Industry Visit**

METHODS OF TEACHING

M4 : Presentation /PPT	**M9 : Project Based**
M5 : Lab/Practical	**M10 : Charts / OHP**

10 QUESTION BANK

Unit I:

1. Mention the advantages of integrated circuits.

2. write down the various processes used to fabricate IC's using silicon planar
3. technology.

4. What is the purpose of oxidation?
5. Why aluminum is preferred for metallization?

6. Define an operational amplifier. Mention the characteristics of an ideal op-amp. Define input offset voltage
7. What are the applications of current sources?
8. Define sensitivity. Mention the advantages of Wilson current source

9. What is a current mirror? Explain the working of a wilder current source
10. What is slew rate? Discuss the methods of improving slew rate.

11. What is an Active load? Explain the CE amplifier with active load
12. Explain pole zero compensation and frequency compensation in op-amp.
13. Define band gap reference? Explain in detail about the reference circuit
14. Briefly explain the method of using constant current bias for increasing CMRR in differential?
15. Explain the operation of a Schmitt trigger circuit
16. Explain the working of full precision rectifier?
17. Define ripple rejection with respect to voltage regulators.

18. With circuit diagram discuss the following applications of op-amp

19. Voltage to current converter(ii)Precision rectifier

20. Explain the operation of a Schmitt trigger circuit

21. Explain the working of full precision rectifier

22. Explain the internal structure of voltage regulator IC 723. Also draw a low
23. voltage Regulator circuit using IC 723andexplain its operation.

24. Explain the following terms in an OP-AMP. Bias current

    a. Thermal drift
    b. Input offset voltage and current
    c. Thermal drift

25. Explain the frequency compensation techniques of OP-AMP

26. Draw the circuit of a symmetrical emitter coupled differential amplifier and derive for CMRR.

27. Write a technical note on frequency response characteristics of differential amplifier. State the importance of frequency compensation

28. What is t instrumentation amplifier? What are the required parameters of an instrumentation amplifier? Explain the working of instrumentation amplifier
29. with neat circuit diagram

30. Explain various DC and AC characteristics of an op.amp. Distinguish between ideal and practical characteristics
31. With circuit and waveforms explain the application of OPAMP as (1)Integrator (2) Voltage series Feedback Compensation

Unit II:

1. Why active filters are preferred?
2. What is meant by cut off frequency of a high pass filter and how it is found out in a first order high pass filter
3. List the applications of 555 timer in monostable mode of operation
4. Define 555 IC?
5. List the basic blocks of IC 555 timer?
6. Define VCO.
7. What does u mean by PLL?
8. List the applications of 565 PLL
9. Define lock range.
10. Define capture range
11. Define pull-in time

Unit III:

1. List the broad classification of ADCs
2. List out the direct type ADCs
3. List out some integrating type converters

4. What is integrating type converter
5. Explain in brief the principle of operation of successive Approximation ADC
6. What are the main advantages of integrating type ADCs
7. What is the main drawback of a dual-slop ADC?
8. Define conversion time.
9. Define accuracy of converter
10. Explain in brief stability of a converter

Unit IV:

1. Explain how PROM, EPROM and EEPROM technologies differ from each other.
2. Design CMOS transistor circuit for 2-input AND gate.
3. Explain sourcing current of TTL output?
4. Which of the parameters decide the fan-out and how?
5. Explain sinking current of TTL output?
6. Explain the term Voltage levels for logic ‘1’ & logic ‘0’ with reference to TTL gate?
7. Explain the DC Noise margin with reference to TTL gate?
8. Explain Low-state unit load with reference to TTL gate?
9. Explain High-state fan-out with reference to TTL gate?
10. Explain the use of Package?

Unit V:

1. Define static RAM
2. Define dynamic RAM
3. Classify types of ROMs
4. Applications of ROMS
5. What is the difference between latch& Flip-Flop, Explain with logic diagram.
6. Explain any one application of SR latch.
7. What is race around condition? how it is avoided?
8. How synchronous counters differ from asynchronous counters?
9. List counter applications.
10. State various applications of counters.

Code No: 07A4EC02

Set No. 1

II B.Tech II Semester Regular Examinations, Apr/May 2009
LINEAR AND DIGITAL IC APPLICATIONS
(Common to Electrical & Electronic Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

*****

1. (a) With help of a block diagram explain the basic building blocks of an Op-amp.
   (b) What does the term 'balanced output' mean in an Op-amp?
   (c) List the parameters that should be considered for AC and DC application. [6+2+8]

2. (a) What is a clipper? With circuit diagram, explain the operation of positive and negative clippers.
   (b) Describe the principle of operation of a precision half wave rectifier with wave forms. [10+6]

3. (a) Derive the expression for frequency of oscillation of a RC phase shift oscillator and explain the operation of the circuit.
   (b) Design a second order low pass filter at a high cut off frequency of 1 KHz. Derive the transfer function of the above filter. [8+8]

4. (a) Give the block diagram of NE 565 PLL and explain the role of each block. Make circuit connections to track the incoming signal and explain its operations.
   (b) With neat sketches, explain the following terms:
      i. Lock-in-range.
      ii. Capture range
      iii. Pull-in time.
   (c) Sketch the capture transient and explain why it is generated before locking? [10+3+3]

5. (a) Differentiate between D-A and A- D CONVERTERS.
   (b) Explain D/A converter with R and 2R resistors. [8+8]

6. (a) Define logic family and explain
   (b) Sketch TTL OR Gate and explain its working
   (c) Sketch TTL AND Gate and explain its working. [4+6+6]

7. Design 8 - bit adder using 7482 . [16]

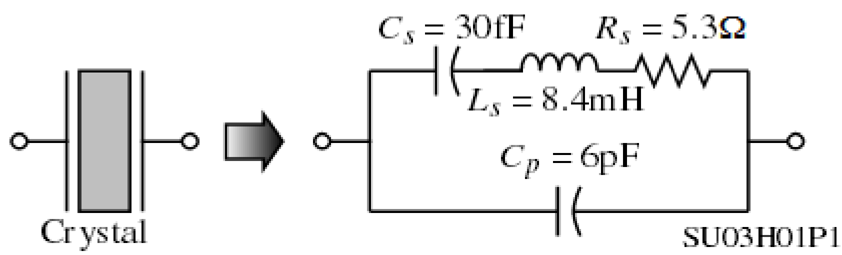
8. (a) What is the major difference between digital and analog PLLs?

Code No: 125EB
JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech III Year I Semester Examinations, May - 2018
LINEAR AND DIGITAL IC APPLICATIONS
(Common to ECE, ETM)
Time: 3 hours Max. Marks: 75
Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B
consists of 5 Units. Answer any one full question from each unit. Each question carries
10 marks and may have a, b, c as sub questions.
PART - A
(25 Marks)
1.a) List the AC characteristics of op-amp. [2]
b) What are the different features of IC 723? [3]
c) What is the significance of VCO in PLL? [2]
d) Compare active and passive filters. [3]
e) What are the applications of ADC? [2]
f) An 8 bit D/A converter as a resolution of 8mV/bit. Find the analog output voltage for
the input 10111010. [3]
g) Which IC is used as BCD code converter? [2]
h) How to drive CMOS gate to TTL gate? [3]
i) How to convert JK flip-flop to D flip flop? [2]
j) List different types of memories. [3]
PART - B
(50 Marks)
2.a) Explain the working of Non-Inverting amplifier and derive the equation of its Gain.
b) How op-amp is used as a differentiator? Explain. [6+4]
OR
3.a) Explain the working of a Schmitt trigger with neat circuit diagram.
b) How op-amp is used for comparator? Explain its working. [5+5]
4.a) Design an active high pass filter with cutoff frequency of 4KHz.
b) How to generate a sawtooth wave form? Explain the working of such a circuit with neat
circuit diagram. [5+5]
OR
5.a) Draw the functional block diagram of 565IC and explain its working.
b) Explain the working of an Astable multivibrator using IC555 with circuit diagram.
[5+5]
6. Explain the working of R-2R ladder DAC with neat circuit diagram and write its
limitations. [10]
OR
7. Explain the working of dual slope ADC with neat circuit diagram and compare its
performance with other ADC. [10]

8. Design a driving circuit for LED and which 74XX series IC is used for it. [10]
OR
9. Design a Priority encoder circuit and which 74XX series IC is used for it. [10]
10. Design a synchronous counter using 74XX ICs and explain its working with neat
timing waveforms. [10]
OR
11. Design a decode counter using Jk-Flip-Flops. [10]
---ooOoo---www.manaresults.co.in

12 TUTORIAL QUESTIONS

1. Explain the Characteristics of 741 IC Along with the Applications? (Essay type)
2. Calculate the LOOP GAINS for 565 IC With different Frequencies?(Problem analysis)
3. Enhance the Op-Amp Performance by Improving Gain and CMRR Ratio?(Research based)
4. Solve for and evaluate the series and parallel resonance frequencies of the crystal Whose model is shown? It is suggested to make appropriate assumptions as the exact frequencies are difficult to achieve? (Problem analysis)



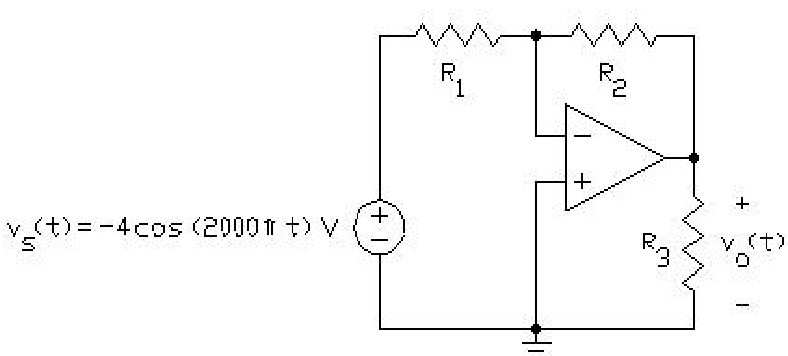
SET-II

1. What is the importance of 555 Timer in IC &Explain how it Works as a Multivibrator?(essay type)
2. Design an R-2R DAC for the input of 1110 and calculate output voltage?(Problem analysis)
3. Design Mobile Phone Detector Using LM358?(Project based)
4. Design a linear PLL in simulink?(Program based)

Assignment II

SET-I

1. Explain the briefly about different types of Combinational circuits?(Essay type)
2. Design a divide by 20 counter using IC 7490?(project based)
3. Analyze OP-AMP circuit using MATLAB?(Program based)



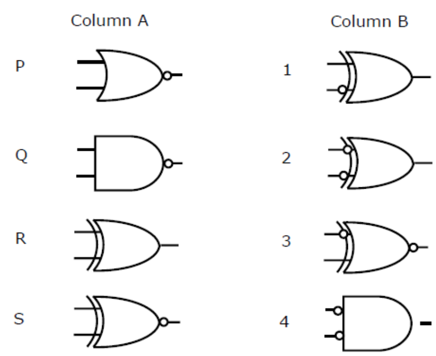
4. Design of OP-AMP using CMOS technology & its application?(research type)

SET-II

1. Using the method of flip-flop conversion carry out the following conversions.(problem analysis)
   i) S-R to T
   ii) J-K to D
   iii) T to D
2. Realize the following expression using 74×151 IC f(Y) = AB + BC + AC.(Problem analysis)
3. Design a Remote Control Light by using 555 Timer?(Project based)
4. To write a VHDL Program to generate a 1010 sequence detector? (Program based)

13 Continuous assessment program (CAP):

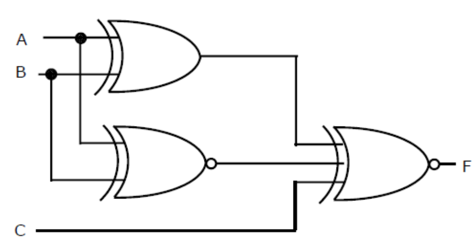
1. Match the logic ga5tes in **Column A** with their equivalents in **Column B.**



(A) P–2, Q-4, R-1, S-3 (B) P-4, Q-2, R-1, S-3

(C) P–2, Q-4, R-3, S-1 (D) P-4, Q-2, R-3, S-1

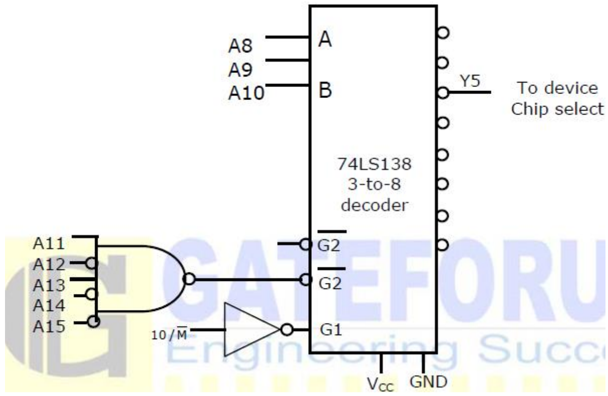
2. For the output F to be 1 in the logic circuit shown, the input combination should be



(A) A = 1, B= 1. C = 0 (B) A = 1, B= 0,C = 0

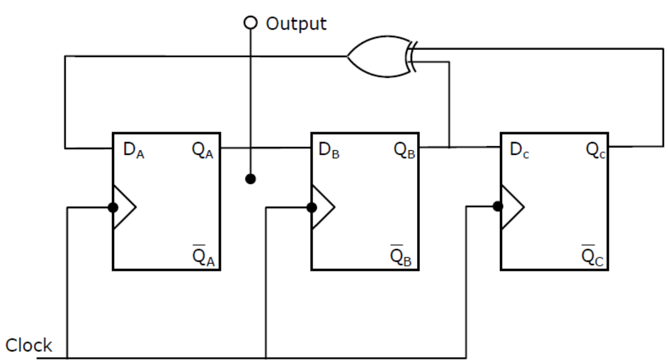
(C) A = 0, B= 1. C = 0 (D) A = 0, B= 0, C = 1

3. In the circuit shown, the device connected to Y5 can have address in the range



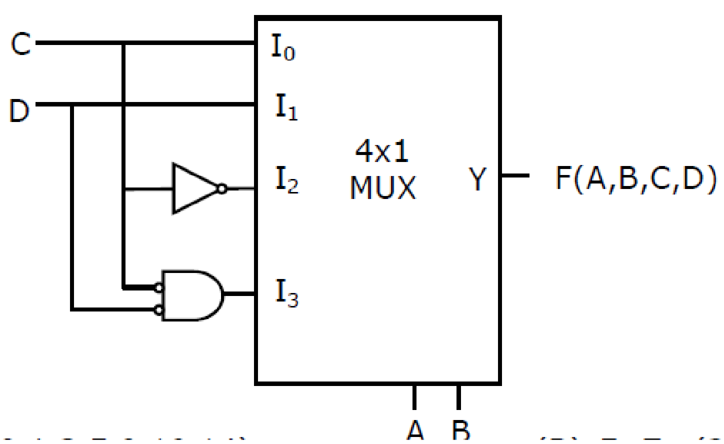
(A) 2000 - 20FF (B) 2D00 – 2DFF (C) 2E00 – 2EFF (D) FD00 - FDFF

4. Assuming that flip-flops are in reset condition initially, the count sequence observed at QA in the circuit shown is



(A) 0010111… (B) 0001011… (C) 0101111… (D) 0110100…

5. The Boolean function realized by the logic circuit shown is

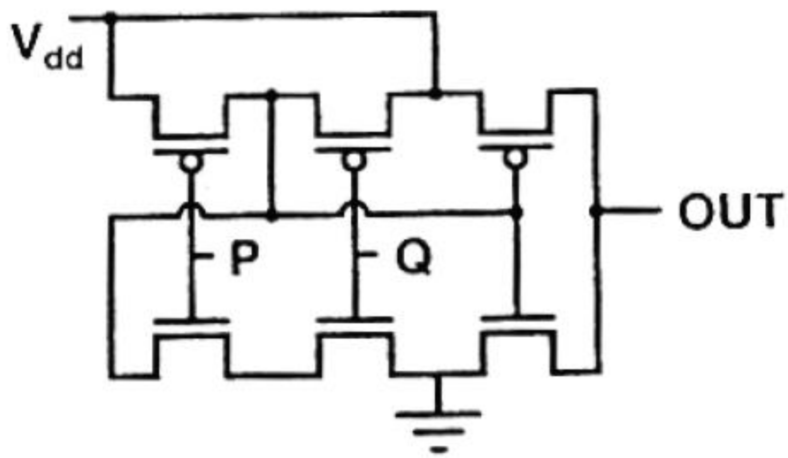


(A) F=Σm(0,1,3,5,9,10,14) (B) F=Σm(2,3,5,7,8,12,13)

(C) F=Σm(1,2,4,5,11,14,15) (D) F=Σm(2,3,5,7,8,9,12)

GATE 2008

6. The logic function implemented by the following circuit at the terminal OUT is



A) P NOR Q B) P NAND Q C) P OR Q D) P AND Q

7 .The two numbers represented in signed 2's complement form are P=11101101 and Q=11100110.If Q is subtracted from P,the value obtained in signed 2's complement form is

A) 100000111 B) 00000111 C) 11111001 D) 111111001

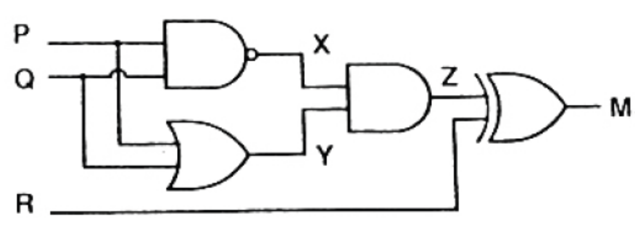
8. Which of the following Boolean Expression correctly represents the relation between P,Q,R and Mi

(a) M = (P OR Q) XOR R

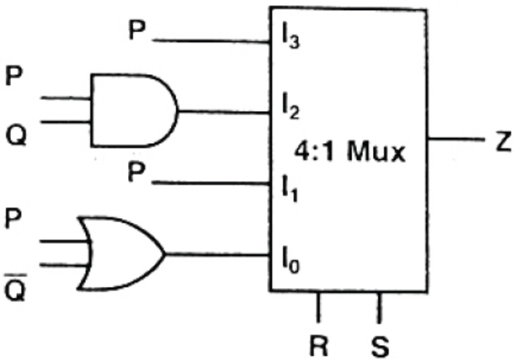
(b) M_1 = (P AND Q) XOR R

(c) M = (P NOR Q) XOR R

(d) M_1 = (P XOR Q) XOR R



9. For the circuit shown in the following figure, Io –I3 are inputs to the 4:1 Multiplexer (MSB) are control bits. The output Z can be represented by



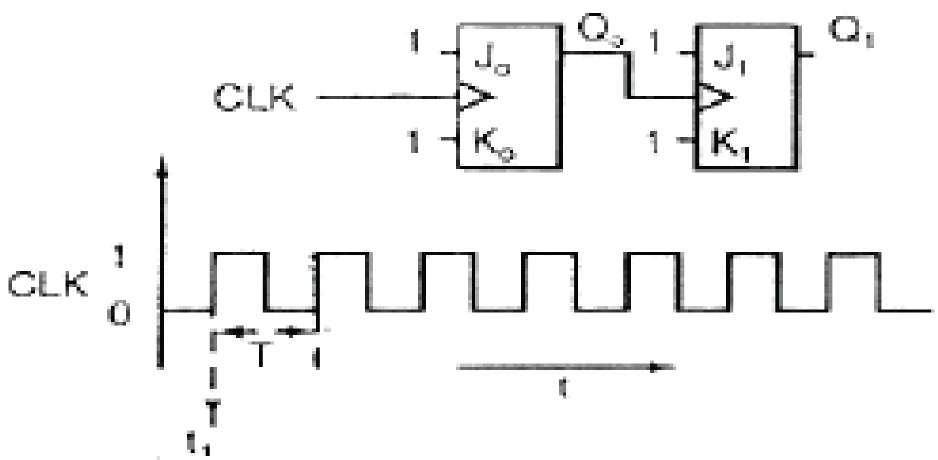
(a) $PQ + P\bar{Q}S + \bar{Q}\bar{R}\bar{S}$

(b) $P\bar{Q} + PQ\bar{R} + \bar{P}\bar{Q}\bar{S}$

(c) $P\bar{Q}\bar{R} + \bar{P}QR + PQRS + \bar{Q}\bar{R}\bar{S}$

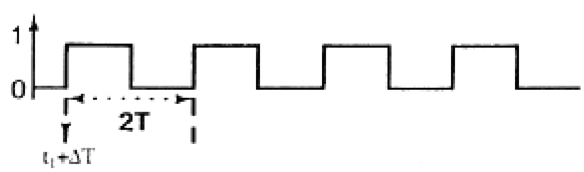
(d) $PQ\bar{R} + PQR\bar{S} - P\bar{Q}\bar{R}S + \bar{Q}\bar{R}\bar{S}$

10.For each of the Positive edge-Triggered J-K flipflop used in the following figure, the propagation delay is ΔT.

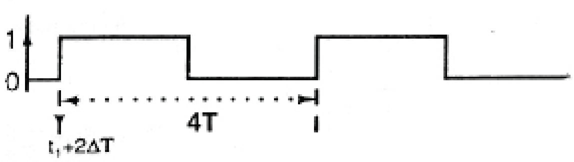


Which of the waveforms correctly represents the output at Q1?

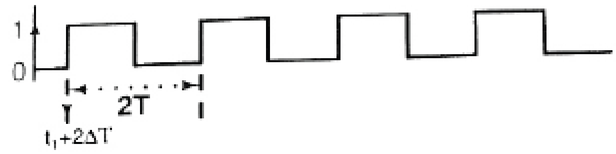
(a)



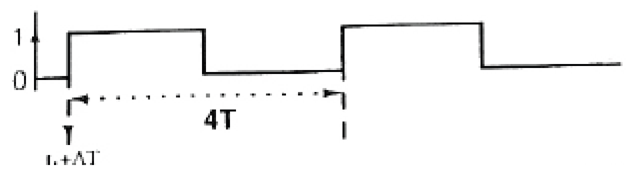
(b)



(c)

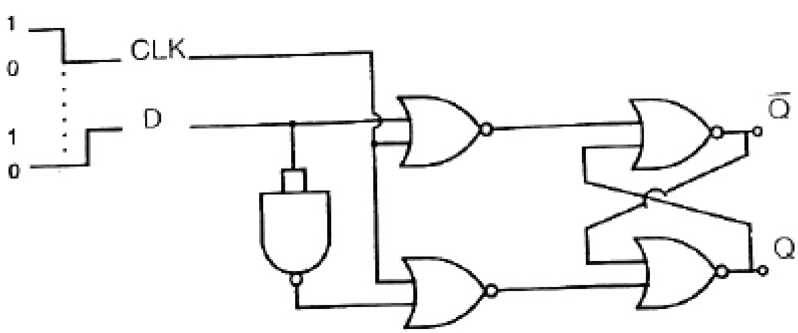


(d)



11. For the circuit shown in the figure D has a transition from 0 to 1 after CLK changes from 1 to 0.

Assume gate delays to be negligible.



Which of the following statement is true?

A) Q goes to 1 at the CLK transition and stays at 1

B) Q goes to 0 at the CLK transition and stays at 0

C) Q goes to 1 at the CLK transition and goes to 0 when D goes to 1

D) Q goes to 0 at the CLK transition and goes to 1 when D goes to 1

GATE 2007

12. X=01110 and Y=11001 are two 5-bit binary numbers represented in Two's complement format. The sum of X and Y represented in Two's complement format using 6 bit is

A) 100111 B) 001000 C) 000111 D) 101001

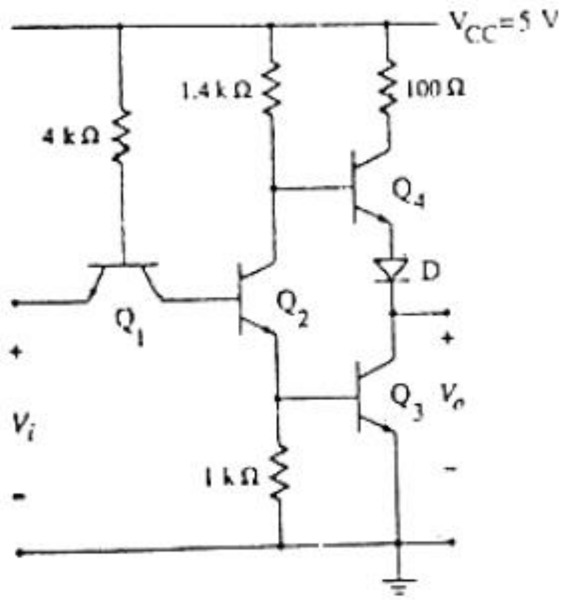
13. The Boolean function Y=AB+CD is to be realized using only 2-input NAND gates. The minimum number of gates required is

A) 2 B) 3 C) 4 D) 5

13. The Boolean Expression Y=A’B’C’D+A’BCD’+AB’C’D+ABC’D’ can be minimized to

A)Y=A’B’C’D+A’BC’+AC’D B)Y=A’B’C’D+BCD’+AB’C’D
C)Y=A’BCD’+B’C’D+AB’C’D D) Y=A’BCD’+B’C’D+ABC’D’

14.The circuit diagram of a standard TTL NOT gate is shown in figure. When V1 =2.5 V,the modes of operation of the transistor will be



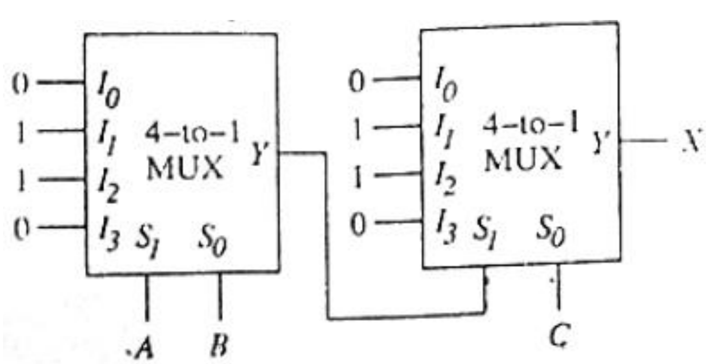
A) Q1: reverse active Q2: normal active Q3: saturation Q4: cut-off

B) Q1: reverse active Q2: saturation Q3: saturation Q4: cut-off

C) Q1: normal active Q2: cut-off Q3: cut-off Q4: saturation

D) Q1: saturation Q2: saturation Q3 saturation: Q4: normal active

15. The following circuit X is given by



a)X=AB’C’+A’BC’+A’B’C+ABC

b) X=A’BC+AB’C+ABC’+A’B’C’

c)X=AB+BC+AC

d) X=A’B’+B’C’+A’C’

SEMINAR TOPICS

1.BCD to gray code converter

2.Digital Oscilloscopes

3.Logic gates emulator

4.The bio chips

5.Air Brake System

(14) - STEP/Course material in softcopy

LDICA.rar

(15) Expert Lectures with topics & Schedules(if any

CMR ENGINEERING COLLEGE

UGC AUTONOMOUS

(Approved by AICTE - New Delhi. Affiliated to JNTUH and Accredited by NAAC & NBA)

CMR ENGINEERING COLLEGE

EXPLORE TO INVENT

NATIONAL BOARD of ACCREDITATION

DEPARTMENT OF

ELECTRONICS AND COMMUNICATION ENGINEERING

ACADEMIC PLANNER 2023-24

COURSE: B-TECH II –II SEM

SUBJECT: NUMERICAL TECHNIQUES AND COMPLEX VARIABLES VARIBLES

ACADEMIC PLANNER

SUBJECT: NUMERICAL TECHNIQUES AND COMPLEX VARIABLES VARIBLES

S.NO		CONTENT
(1)	-	Preamble/ Introduction
(2)	-	Prerequisites
(3)	-	Objectives and Outcomes
(4)	-	Syllabus
		1.JNTU
		2.GATE
		3.IES
(5)	-	List of Expert Details
(6)	-	Journals
(7)	-	Subject-Lesson Plan
(8)	-	Suggested Books
(9)	-	Websites for Self Learning
(10)	-	Question Banks
		1.JNTU/Model Papers
		2.GATE
(11)	-	Two Case Study Presentations
(12)	-	Assignment Questions/Innovative
Assignments Sets		
(13)	-	List of topics for student's seminars
(14)	-	STEP/Course Material
(15)	-	Expert Lectures with Topics & Schedules

(1) ***PREAMBLE/ INTRODUCTION***

This subject provides an insight for the students to improve their ability to think critically, to analyze a real problem and solve it using a wide array of mathematical tools. It also helps the students to apply these ideas to a wide range of problems that include the Engineering applications.

(2) *PREREQUISITES*

Mathematical Knowledge at pre-university level.

(3) *OBJECTIVES AND OUTCOMES*

I. COURSE OBJECTIVES:

To learn concepts, properties of Laplace transforms

20. Various numerical methods to find roots of polynomial and transcendental equations.
21. Concept of finite differences and to estimate the value for the given data using interpolation.
22. Evaluation of integrals using numerical techniques.
23. Solving ordinary differential equations of first order using numerical techniques.
24. Differentiation and integration of complex valued functions.
25. Evaluation of integrals using Cauchy's integral formula and Cauchy's residue theorem.
26. Expansion of complex functions using Taylor's and Laurent's series.
27. Expressing periodic function by Fourier series and a non-periodic function by Fourier transforms.

II. COURSE OUTCOMES:

After learning the contents of this paper the student must be able to

S. No	Description	Bloom's Taxonomy Level
1.	Find the root of a given polynomial and transcendental equations and estimate the value for the given data using interpolation.	Understand, Apply (Level 2, Level 3)
2.	Find the numerical solutions for a given first order ODE's and integrations.	Understand, Apply (Level 2, Level 3)

3.	Analyze the complex function with reference to their analyticity	Apply, Create (Level 4, Level 6)
4.	Evaluate complex integration using Cauchy's integral and residue theorem	Analyze, Apply (Level 4, Level 3)
5.	Express any periodic function in terms of sine and cosine	Understand, Apply (Level 2, Level 3)

(4.1) *SYLLABUS – JNTU*

UNIT-I: Numerical Methods-I **(10 hours)**

Bisection method, Iteration Method, Newton - Raphson method and Regula-Falsi method, Gauss Seidel method for solving linear system of equations. Finite differences: Forward differences-Backward Differences-Central differences. Newton's forward and backward difference formulae. Central difference interpolation: Gauss's forward and backward formulae; Lagrange's Interpolation formula.

UNIT-II: Numerical Methods-II **(8 hours)**

Numerical Integration: Trapezoidal rule, Simpson's 1/3 Rule, Simpson's 3/8 Rule.

Ordinary differential equations: Taylor's series, Picard's method, Euler's method,Runge-Kutta 2^{nd} and 4^{th} order methods.

UNIT-III: Complex Differentiation **(10 hours)**

Limit, Continuity and Differentiation of Complex functions. Cauchy-Riemann equations, Milne-Thomson method, analytic functions, harmonic functions, finding harmonic conjugate.

UNIT – IV: Complex Integration **(10 hours)**

Line integrals, Cauchy's theorem, Cauchy's Integral formula, zeros of analytic functions, singularities, Taylor's series, Laurent's series, Residues, Cauchy's Residue theorem.

UNIT – V: Fourier Series & Fourier Transforms **(10 hours)**

Fourier series - Dirichlet's Conditions - Half-range Fourier series - Fourier Transforms: Fourier Sine and cosine transforms - Inverse Fourier transforms.

TEXT BOOKS:

1. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36^{th} Edition, 2010.
2. Erwin Kreyszig, Advanced Engineering Mathematics, 9^{th} Edition, John Wiley & Sons, 2006.
3. B.V.Ramana, A text Book of Engineering Mathematics, Tata Mc Graw Hill.

REFERENCES:

1. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9th Edition, Pearson, Reprint, 2002.
2. Shahnaz Bathul, A text book of Mathematical Methods, Right Publishers.

(4.2) *SYLLABUS - GATE*

Complex Analysis: Analytic functions, Cauchy's integral theorem, Cauchy's integral formula; Taylor's and Laurent's series, residue theorem.

Numerical Methods: Solution of nonlinear equations, single and multi-step methods for differential equations, convergence criteria.

(4.3) *SYLLABUS - IES*

Complex Analysis: Analytic functions, Cauchy's integral theorem, Cauchy's integral formula; Taylor's and Laurent's series, residue theorem.

Numerical Methods: Solution of nonlinear equations, single and multi-step methods for differential equations, convergence criteria.

(5) *LIST OF EXPERT DETAILS*

The Expert Details which have been mentioned below are only a few of the eminent ones known Internationally, Nationally and Locally. There are a few others known as well.

INTERNATIONAL

Prof. Gary A. Lorden,
Research Area:Probability and Statistics, Postal
Address: Department of Mathematics California
Institute of Technology, California
Email ID: glorden@caltech.edu

Assoc. Prof. Daniel C. Weiner
Research Area:Probability, Randomness, Data analysis and Statistics Postal Address: Department of Mathematics.,Boston University, Boston, Massachusetts,
Email ID: weiner@bu.edu

NATIONAL

Prof. P. Vellaisamy

Research Area: Applicable Mathematics, Statistics and Probability,

Postal Address: Department of Mathematics,IIT Mumbai.

Email ID : pv@math.iitb.ac.in

Dr.Neelesh. S. Upadhye
Research Area : Applied Probability,
Postal Address: Department of Mathematics,IIT,Chennai.
Email ID: neelesh@iitm.ac.in

REGIONAL

Dr. Y. Rameshwar
Assistant Professor Department of Mathematics, University College of Science, OU, Saifabad, Hyderabad – 500004.
Email ID: rameshwar@osmania.ac.in

Dr. M.V. Ramana Murthy
Professor & HOD, MGIT, Hyderabad – 500075.
Email ID: mv.rm50@gmail.com

(6) *JOURNALS*

INTERNATIONAL

1. Journal of American Mathematical Society
2. Journal of differential equations - Elsevier
3. Pacific Journal of Mathematics
4. Journal of Australian Society
5. Bulletin of "The American Mathematical Society"
6. Bulletin of "The Australian Mathematical Society"
7. Bulletin of "The London Mathematical Society"

NATIONAL

1. Journal of Interdisciplinary Mathematics
2. Indian Journal of Pure and Applied Mathematics
3. Indian Journal of Mathematics
4. Proceedings of Mathematical Sciences
5. Journal of Mathematical and Physical Sciences.
6. Journal of Indian Academy and Sciences

(7) *SUBJECT-LESSON PLAN*

Subject code	Name of the subject	Year/Branch	Name of the Faculty
MA401BS	NUMERICAL TECHNIQUES AND COMPLEX VARIABLES VARIBLES	II B.TECH II SEM ECE	1) A.SREEHARI

S.NO	Sub-Topic	NO. OF LECTURES REQUIRED	PLANNED DATE	CONDUCTEDDATE	Remarks
UNIT-I Numerical Methods-I					
1	Intoduction	L1	**19/2/2024**	**19/2/2024**	
2	Bisection method	L2,L3	**20/2/2024 to 21/2/2024**	**20/2/2024 to 21/2/2024**	
3	Regual falsi method	L4,L5,L6	**24/2/2024 to 26/2/2024**	**24/2/2024 to 26/2/2024**	
4	N-R method	L7	**28/2/2024**	**28/2/2024**	
5	Iteration method	L8	**2/3/2024**	**2/3/2024**	
6	Newtons forward method	L9	**4/3/2024**	**4/3/2024**	
7	Newtons backword method	L10	**5/3/2024**	**5/3/2024**	

8	Interpolatio n	L11,L12	**6/3/2024-9/3/2024**	**6/3/2024-9/3/2024**	
9	Gauss seidal method	L13-L14	**11/3/2024 – 13/3/2024**	**11/3/2024 – 13/3/2024**	
UNIT-II Numerical Methods-II					
10	Numerical Integration	**L15**	**16/3/2024**		
11	Numerical Integration	**L16-L18**	**18/3/2024-20/3/2024**		
12	Taylors method	**L19-L20**	**22/3/2024 – 23/3/2024**		
13	Taylors method	**L21**	**26/3/2024**		
14	Picard's Method	**L22-23**	**2/3/2024 – 27/3/2024**		
15	Euler's method R-K Method	**L24**	**29/3/2024**		
16	Modified Euler's method	**L25**	**30/3/2024**		
17	Modified Euler's method	**L26**	**30/3/2024**		
18	R-K Method	**L27-L28**	**1/4/2024 – 3/4/2024**		
19	R-K Method	**L29**	**6/4/2024**		
20	Practice	**L30**	**6/4/2024**		
21	Practice	**L31**	**8/4/2024**		
UNIT-III Complex Differentiation					

22	Limits , Continuity	**L32**	**10/4/2024**		
23	Differentiation of Complex Functions	**L33**	**22/4/2024**		
24	C-R Equations	**L34**	**23/4/2024**		
25	Harmonic Functions & Harmonic Conjugate	**L35**	**28/4/2024**		
26	Milne - Thomson Method	**L36**	**1/5/2024**		
			UNIT-IV **Complex Integration**		
27	Line Integrals	**L37**	**4/5/2024**		
28	Cauchy's Theorem	**L38**	**6/5/2024**		
29	Cauchy's Integral Formula , Generalized Cauchy's Integral Formula for Derivatives	**L39**	**7/5/2024**		
30	Taylor's Series , Laurent's Series	**L40**	**8/5/2024**		

31	Zeros of analytical functions	**L41**	**9/5/2024**		
32	Singularities & its types, Poles , Residues	**L42**	**11/5/2024**		
33	Cauchy's Residue Theorem	**L43**	27/5/2024		
UNIT-V Fourier Series & Fourier Transforms					
34	Dirichlet’s Conditions	L44	28/5/2024		
35	Half-range Fourier series,	L45-L46,L-47	29/5/2024 – 30/5/2024, 3/6/2025		
36	Fourier Transforms	L48-L49-L50	4/6/2024 – 5/6/2024, 7/6/2025		
37	Fourier Sine and cosine transforms	L51,L52,L53	8/6/2024, 11/6/2024,13/6/2024,		
38	Inverse Fourier transforms	L54,L55,L56	14/6/2024, 18/6/2024,19/6/2024		
39	Practice	L57	20/6/2024		
4-	Practice	L58	22/6/24		

(9) *WEBSITESFOR SELF LEARNING*

12. https://www.youtube.com/watch?v=EDVJotmT584
13. https://www.youtube.com/watch?v=qhUIx096afA
14. https://www.youtube.com/watch?v=iviiGB5vxLA
15. https://www.youtube.com/watch?v=t9xW7UaZwZ0&list=PLU6SqdYcYsfI3sh-ho_iiTkCGsTbVh_Sw
16. https://www.youtube.com/watch?v=ywQVarOaA60
17. https://nptel.ac.in/courses/111/106/111106139/#

(10.1) *QUESTION BANKS - JNTU*

UNIT- I

<table>
<tr><th>S.No</th><th>Question</th><th>Blooms taxonomy level</th><th>Course outcome</th></tr>
<tr><td>1</td><td>State Newton’s forward difference formula for equal intervals.</td><td>Apply</td><td>2</td></tr>
<tr><td>2</td><td>Find the divided differences of $f(x) = x3 - x2 + 3x + 8$for the arguments 0,1,4,5.</td><td>Evaluate</td><td>2</td></tr>
<tr><td>3</td><td>Construct a table of divided difference for the given data:
<table>
<tr><td>X</td><td>54</td><td>58</td><td>59</td><td>61</td></tr>
<tr><td>y</td><td>2.81</td><td>2.82</td><td>2.81</td><td>2.82</td></tr>
</table></td><td>Apply</td><td>2</td></tr>
<tr><td>4</td><td>Write down the Newton’s forward difference interpolation formula for equal intervals</td><td>Evaluate</td><td>2</td></tr>
<tr><td>5</td><td>State Newton’s forward formula and Backward formula.</td><td>Understand</td><td>2</td></tr>
<tr><td>6</td><td>Construct the divided difference table for the data (0, 1), (1, 4) , (3, 40) and (4, 85).</td><td>Understand</td><td>2</td></tr>
<tr><td>7</td><td>Given $y0 = 3$, $y1 = 12$, $y2 = 81$, $y3 = 200$, $y4 = 100$. Find$\Delta^4 y0$.</td><td>Apply</td><td>2</td></tr>
<tr><td>8</td><td>Define Δ, ∇, E.</td><td>Understand</td><td>2</td></tr>
<tr><td>9</td><td>What are the n^{th} divided differences of a polynomial of the n^{th} degree?</td><td>Apply</td><td>2</td></tr>
<tr><td>10</td><td>Using divided differences, show that $f(x, x) = f'(x)$ through the limiting process.</td><td>Apply</td><td>2</td></tr>
</table>

UNIT - II

S.No	Question	Blooms taxonomy level	Course outcome
1	State Trapezoidal rule to evaluate a double integral.	Apply	3
2	State Simpson's 1/3 rule to evaluate a double integral.	Apply	3
3	what is the order of truncation error in Taylor's series method of nth order?	Apply	3
4	Compare Taylor series and Runge-Kutta methods.	Apply	3
5	What do we mean by single step methods and multistep methods?	Apply	3
6	Why the predictor-corrector methods are called so?	Apply	3
7	Compare Runge-Kutta method and predictor corrector methods	Apply	3
8	Write down the Milne's predictor-corrector formula	Apply	3
9	State Simpson's 1/3 rule to evaluate a double integral.	Apply	3
10	State Simpson's 1/3 and 3/8 formulas for numerical integration.	Apply	3

UNIT III

S.No	Question	Blooms taxonomy level	Course outcome
1	If w = log z, find $\frac{dw}{dz}$ and determine where w is not-analytic.	Understand	4
2	Every analytic function f(z) = u+iv defines two families c curves u(x,y) =k1 and v(x,y) = k2 forming an Orthogonal s	Understand	4
3	Evaluate $\int(z^2+3z)dz$ along the straight line from (2,0) to (2,2) and then from (2,2) to (0,2).	Evaluate	4
4	Find the analytic function of $f(z)$ □ u □ iv *if* u □ v □ $e^x(\cos y$ □ $\sin y)$	Apply	4
5	S.T $u(x, y) = e^{2x}(x \cos 2y - y \sin 2y)$ is Harmonic and find its Harmonic conjugate	Apply	4
6	Show that the both Real and Imaginary parts of an Analytic function are Harmonic.	Apply	4
7	Derive the polar form of Cauchy Riemann equation	Evaluate	4
8	Given $u(x, y) = e^{2x}(x \cos 2y - y \sin 2y)$, find its Analytic Function	Apply	4
9	Find the analytic function $f(z) = u+iv$ if $u-v = (\cos y - \sin y)$ find f(z) in terms of z.	Apply	4
10	Show that f(x,y) = |xy| is analytic except at origin.	Apply	4

UNIT- IV

S.No	Question	Blooms taxonomy level	Course outcome
1	Expand $f(z) = \int \frac{1}{z^2-3z+2}$ in the region $1<\|z\|<2$ by Laurent's series.	Apply	5
2	Verify Cauchy's theorem, for $\int Z^3\, dz$, taken over the boundary of the rectangle with vertices -1, 1, (1+i), (-1+i)	Apply	5
3	Expand *sinhz* by Taylor's series about $z=\pi i$	Apply	5
4	Evaluate $\oint \frac{4z-3}{z(z-1)(z-2)}dz$, where C is the circle $\|Z$ = e^x using Residue Theorem	Apply	5

5	$\oint_c \frac{e^{2z}}{(z-1)(z-2)} dz$, *where c is the circle* $\lvert z \rvert = 3$ by Cauchy residue theorem	Apply	5
6	Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ if C is the square with vertices (1 + i) (1 – i) (-1 + i) (-1- i)	Apply	5
7	Determine the poles of the function $\frac{z^2}{(z-1)^2(z+2)}$ and the Residues at each pole.	Apply	5
8	Find the fixed points of the transformation (i) $W = \frac{6z-9}{z}$ (ii) $W = \frac{z-i}{z+i}$	Apply	5
9	Find the Mobius transformation that maps the point (-1,0,1) into the points (0,i,3i)	Apply	5
10	Find the Bilinear transformations which maps (0,1,∞) to (- 1, - 2, -I)	Apply	5

Code No: 154BG

R18

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year II Semester Examinations, November/December - 2020

LAPLACE TRANSFORMS, NUMERICAL METHODS AND COMPLEX VARIABLES

(Common to EEE, ECE, EIE)

Time: 2 hours **Max. Marks: 75**

Answer any Five Questions

All Questions Carry Equal Marks

- - -

1.a) Solve the differential equation using Laplace transforms $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = e^{-t}; x(0) = 0, x'(0) = 1$.

b) Prove that $L^{-1}\{F(s)\} = f(t)$ and $f(0) = 0$ then $L^{-1}\{sF(s)\} = \frac{df}{dt}$ [10+5]

2.a) Find up to the four places of decimals the smallest root of the equation $e^{-x} = \sin x$ using Newton-Raphson method.

b) Fit a polynomial of second degree to the data points (2,3.07), (4,12.85), (6,31.47), (8,57.38) and (10,91.29).

c) Find the root of the equation $x \log_{10} x = 1.2$ using False position method. [5+5+5]

3. Evaluate the integral $\int_0^1 \frac{dx}{3+2x}$ using trapezoidal rule. [15]

4. Use Runge-Kutta method of order four to find y when $x = 0.6$ given that $\frac{dy}{dx} = 1 + y^2, y(0) = 0$. [15]

5. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$. Compute $y(0.1)$ in steps of 0.02 using Euler's modified method. [15]

6.a) Prove that the function $f(z) = \sqrt{xy}$ is not analytic at the origin even though the C-R equations are satisfied there at.

b) If $f(z) = u + iv$ is an analytic function in a region R, prove that the curves $u(x, y) = c_1$, $v(x, y) = c_2$ form two orthogonal families. [8+7]

7.a) Evaluate $\oint_C \frac{e^z}{(z+1)^2} dz$, where C is the circle $|z - 3| = 3$.

b) Find the residue of $e^z \operatorname{cosec}^2 z$. [8+7]

8. Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices $-1, 1, 1+i, -1+i$. [15]

---ooOoo---

$$\frac{1}{ms^2 + \gamma s + k} = \varphi(t).$$

Now, impulse inputs are usually modeled in terms of delta functions. Thus, knowing the Laplace transform of such functions is important when solving differential equations. The next theorem finds the Laplace transform of the delta function.

Theorem 48.1
With $\delta(t)$ defined as above, if $a \leq t_0 < b$

$$\int_a^b f(t)\delta(t - t_0)dt = f(t_0).$$

Proof.
We have

$$\begin{aligned}
\int_a^b f(t)\delta(t-t_0) &= \lim_{\epsilon\to 0^+}\int_a^b f(t)p_\epsilon(t-t_0)dt \\
&= \lim_{\epsilon\to 0^+}\frac{1}{\epsilon}\int_{t_0}^{t_0+\epsilon} f(t)dt \\
&= \lim_{\epsilon\to 0^+}\frac{1}{\epsilon}f(t_0+\beta\epsilon)\epsilon = f(t_0)
\end{aligned}$$

where $0 < \beta < 1$ and the mean-value theorem for integrals has been used ■

Remark 48.1
Since $p_\epsilon(t-t_0) = \frac{1}{\epsilon}$ for $t_0 \leq t \leq t_0+\epsilon$ and 0 otherwise we see that $\int_a^b f(t)\delta(t-a)dt = f(a)$ and $\int_a^b f(t)\delta(t-t_0)dt = 0$ for $t_0 \geq b$.

It follows immediately from the above theorem that

$$L[\delta(t-t_0)] = \int_0^\infty e^{-st}\delta(t-t_0)dt = e^{-st_0}, \quad t_0 \geq 0.$$

In particular, if $t_0 = 0$ we find

$$L[\delta(t)] = 1.$$

The following example illustrates the formal use of the delta function.

Example 48.1
A spring-mass system with mass 2, damping 4, and spring constant 10 is subject to a hammer blow at time $t = 0$. The blow imparts a total impulse of 1 to the system, which was initially at rest. Find the response of the system.

Solution.

The situation is modeled by the initial value problem

$$2y'' + 4y' + 10y = \delta(t), \quad y(0) = 0, \quad y'(0) = 0.$$

Taking Laplace transform of both sides we find

$$2s^2Y(s) + 4sY(s) + 10Y(s) = 1.$$

Solving for $Y(s)$ we find

$$Y(s) = \frac{1}{2s^2 + 4s + 10}.$$

The impulsive response is

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{2}\frac{1}{(s+1)^2 + 2^2}\right] = \frac{1}{4}e^{-2t}\sin 2t \quad \blacksquare$$

Example 48.2

A 16 lb weight is attached to a spring with a spring constant equal to 2 lb/ft. Neglect damping. The weight is released from rest at 3 ft below the equilibrium position. At $t = 2\pi$ sec, it is struck with a hammer, providing an impulse of 4 lb-sec. Determine the displacement function $y(t)$ of the weight.

Solution.

This situation is modeled by the initial value problem

$$\frac{16}{32}y'' + 2y = 4\delta(t - 2\pi),\ y(0) = 3,\ y'(0) = 0.$$

Apply Laplace transform to both sides to obtain

$$s^2Y(s) - 3s + 4Y(s) = 8e^{-2\pi s}.$$

Solving for $Y(s)$ we find

$$Y(s) = \frac{3s}{s^2 + 4} + \frac{e^{-2\pi s}}{s^2 + 4}.$$

Now take the inverse Laplace transform to get

$$y(t) = \mathcal{L}^{-1}[Y(s)] = 3\cos 2t + 8h(t - 2\pi)f(t - 2\pi)$$

where

$$f(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2 + 4}\right] = \frac{1}{2}\sin 2t.$$

Hence,

$y(t) = 3\cos 2t + 4h(t - 2\pi)\sin 2(t - 2\pi) = 3\cos 2t + 4h(t - 2\pi)\sin 2t$ or more explicitly

$$y(t) = \begin{cases} 3\cos 2t, & t < 2\pi \\ 3\cos 2t + 4\sin 2t, & t \geq 2\pi \end{cases} \blacksquare$$

Practice Problems

Problem 48.1

Evaluate

(a) $\int_0^3 (1+e^{-t})\delta(t-2)dt.$

$\frac{1}{2}(1+e^{-t})\delta(t-2)dt.$

2 cos $2t$ $\delta(t)dt.$

te^{-t}

(a) $\frac{2}{1}$ $\delta(t+2)$

$\int_2$

(b) $(e^{2t}+t)$ $\delta(t$ $1)$ $dt.$

-1

$\delta(t-3)$

Problem 48.2

Let $f(t)$ be a function defined and continuous on $0 \le t < \infty$. Determine

$$(f*\delta)(t) = \int_0^t f(t \quad s)\delta(s)ds.$$

Problem 48.3

Determine a value of the constant t_0 such that $\int_0^1 \sin^2[\pi(t-t_0)]\delta(t-\frac{1}{2})dt = \frac{3}{4}$.

Problem 48.4

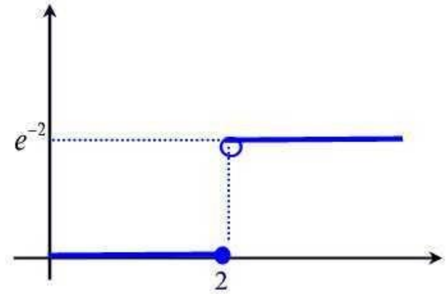
If $\int_1^5 t^n\delta(t-2)dt = 8$, what is the exponent n?

Problem 48.5

Sketch the graph of the function $g(t)$ which is defined by $g(t) = \int_0^t \int_0^s \delta(u-1)duds,\ 0 \le t < \infty.$

Problem 48.6

Determine the constants α 0 0



The graph of the function $g(t) = \int^{t} e^{\alpha t}\delta(t - t\ )dt, \quad 0 \leq t < \infty$ is shown.

Problem 48.7

(a) Use the method of integarting factor to solve the initial value problem $y' \quad y = h(t),\ y(0) = 0.$

(b) Use the Laplace transform to solve the initial value problem $\varphi' \quad \varphi = \delta(t),\ \varphi(0) = 0.$

(c) Evaluate the convolution $\varphi \ h(t)$ and compare the resulting function with the solution obtained in part(a).

Problem 48.8

Solve the initial value problem

$$y' + y = 2 + \delta(t-1), \quad y(0) = 0, \quad 0 \le t \le 6.$$

Graph the solution on the indicated interval.

Problem 48.9

Solve the initial value problem

$$y'' = \delta(t-1) - \delta(t-3), \quad y(0) = 0, \quad y'(0) = 0, \quad 0 \le t \le 6.$$

Graph the solution on the indicated interval.

Problem 48.10

Solve the initial value problem

$$y'' - 2y' = \delta(t-1), \quad y(0) = 1, \quad y'(0) = 0, \quad 0 \le t \le 2.$$

Graph the solution on the indicated interval.

Problem 48.11

Solve the initial value problem

$$y'' + 2y' + y = \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1, \quad 0 \le t \le 6.$$

Graph the solution on the indicated interval.

49 Solving Systems of Differential Equations Using Laplace Transform

In this section we extend the definition of Laplace transform to matrix-valued functions and apply this extension to solving systems of differential equations. Let $y_1(t), y_2(t), \cdots, y_n(t)$ be members of PE. Consider the vector-valued function

$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{bmatrix}$$

The Laplace transform of $\mathbf{y}(t)$ is

$$\begin{aligned} L[\mathbf{y}(t)] &= \int_0^\infty \mathbf{y}(t)e^{-st}dt \\ &= \begin{bmatrix} \int_0^\infty y_1(t)e^{-st}dt \\ \int_0^\infty y_2(t)e^{-st}dt \\ \vdots \\ \int_0^\infty y_n(t)e^{-st}dt \end{bmatrix} \\ &= \begin{bmatrix} L[y_1(t)] \\ L[y_2(t)] \\ \vdots \\ L[y_n(t)] \end{bmatrix} \end{aligned}$$

In a similar way, we define the Laplace transform of an $m \times n$ matrix to be the $m \times n$ matrix consisting of the Laplace transforms of the component functions. If the Laplace transform of each component exists then we say $\mathbf{y}$(t) is **Laplace transformable.**

Example 49.1

Find the Laplace transform of the vector-valued function

$$\mathbf{y}(t) = \begin{matrix} t^2 \\ 1 \\ e^t \end{matrix}$$

Solution.

The Laplace transform is

$$\mathcal{L}[\mathbf{y}(t)] = \begin{bmatrix} \frac{6}{s^3} \\ \frac{1}{s} \\ \frac{1}{s-1} \end{bmatrix}, \quad s > 1 \quad \blacksquare$$

The linearity property of the Laplace transform can be used to establish the following result.

Theorem 49.1

If $\mathbf{A}$ is a constant $n \times n$ matrix and $\mathbf{B}$ is an $n \times p$ matrix-valued function then

$$\mathcal{L}[\mathbf{A}\mathbf{B}(t)] = \mathbf{A}\mathcal{L}[\mathbf{B}(t)].$$

Proof.

Let $\mathbf{A} = (a_{ij})$ and $\mathbf{B}(t) = (b_{ij}(t))$. Then $\mathbf{A}\mathbf{B}(t) = (\sum_{k=1}^{n} a_{ik}b_{kp})$. Hence,

$$\mathcal{L}[\mathbf{A}\mathbf{B}(t)] = [\mathcal{L}(\sum_{k=1}^{n} a_{ik}b_{kp})] = [\sum_{k=1}^{n} a_{ik}\mathcal{L}(b_{kp})] = \mathbf{A}\mathcal{L}[\mathbf{B}(t)] \quad \blacksquare$$

Theorem 42.3 can be extended to vector-valued functions.

Theorem 49.2

(a) Suppose that $\mathbf{y}(t)$ is continuous for $t \geq 0$ and let the components of the derivative vector $\mathbf{y}'$ be members of PE. Then

$$\mathbf{L}[\mathbf{y}'(t)] = s\mathbf{L}[\mathbf{y}(t)] - \mathbf{y}(0).$$

(b) Let $\mathbf{y}'(t)$ be continuous for $t \geq 0$, and let the entries of $\mathbf{y}''(t)$ be membersof PE. Then

$$\mathbf{L}[\mathbf{y}''(t)] = s^2\mathbf{L}[\mathbf{y}(t)] - s\mathbf{y}(0) - \mathbf{y}'(0).$$

(c) Let the entries of $\mathbf{y}(t)$ be members of PE. Then

$$\int_0^t \mathbf{y}(s)ds = \frac{\mathbf{L}[\mathbf{y}(t)]}{s}.$$

Proof.

(a) We have

$$\mathbf{L}[\mathbf{y}'(t)] = \begin{bmatrix} \mathbf{L}[y_1'(t)] \\ \mathbf{L}[y_2'(t)] \\ \vdots \\ \mathbf{L}[y_n'(t)] \end{bmatrix}$$

$$= \begin{bmatrix} s\mathbf{L}[y_1(t)] - y_1(0) \\ s\mathbf{L}[y_2(t)] - y_2(0) \\ \vdots \\ s\mathbf{L}[y_n(t)] - y_n(0) \end{bmatrix}$$

$$= s\mathbf{L}[\mathbf{y}(t)] - \mathbf{y}(0)$$

(b) We have

$$\begin{aligned} \mathbf{L}[\mathbf{y}''(t)] &= s\mathbf{L}[\mathbf{y}'(t)] - \mathbf{y}'(0) \\ &= s(s\mathbf{L}[\mathbf{y}(t)] - \mathbf{y}(0)) - \mathbf{y}'(0) \\ &= s^2\mathbf{L}[\mathbf{y}(t)] - s\mathbf{y}(0) - \mathbf{y}'(0) \end{aligned}$$

(c) We have

$$\mathbf{L}[\mathbf{y}(t)] = s\mathbf{L}\left[\int_0^t \mathbf{y}(s)ds\right]$$

so that

$$\mathbf{L}\left[\int_0^t \mathbf{y}(s)ds\right] = \frac{\mathbf{L}[\mathbf{y}(t)]}{s} \qquad \blacksquare$$

The above two theorems can be used for solving the following initial value problem

$$\mathbf{y}'(t) = \mathbf{A}\mathbf{y} + \mathbf{g}(t), \quad \mathbf{y}(0) = \mathbf{y}_0, \quad t > 0 \tag{8}$$

where $\mathbf{A}$ is a constant matrix and the components of $\mathbf{g}(t)$ are members of

.

Using the above theorems we can write

$$s\mathbf{Y}(s) - \mathbf{y}_0 = \mathbf{A}\mathbf{Y}(s) + \mathbf{G}(s)$$

or

$$(s\mathbf{I} - \mathbf{A})\mathbf{Y}(s) = \mathbf{y}_0 + \mathbf{G}(s)$$

where $\mathcal{L}[\mathbf{g}(t)] = \mathbf{G}(s)$. If s is not an eigenvalue of $\mathbf{A}$ then the matrix $s\mathbf{I} - \mathbf{A}$ is invertible and in this case we have

$$\mathbf{Y}(s) = (s\mathbf{I} - \mathbf{A})^{-1}[\mathbf{y}_0 + \mathbf{G}(s)]. \tag{9}$$

To compute $\mathbf{y}(t) = \mathcal{L}^{-1}[\mathbf{Y}(s)]$ we compute the inverse Laplace transform of each component of $\mathbf{Y}(s)$. We illustrate the above discussion in the next example.

Example 49.2

Solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{y} + \begin{bmatrix} e^{2t} \\ -2t \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Solution.

We have

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{(s+1)(s-3)} \begin{bmatrix} s-1 & 2 \\ 2 & s-1 \end{bmatrix}$$

and

$$\mathbf{G}(s) = \begin{bmatrix} \frac{1}{s-2} \\ -\frac{2}{s^2} \end{bmatrix}.$$

Thus,

$$\begin{aligned} \mathbf{Y}(s) &= (s\mathbf{I} - \mathbf{A})^{-1}[\mathbf{y}_0 + \mathbf{G}(s)] = \frac{1}{(s+1)(s-3)} \begin{bmatrix} s-1 & 2 \\ 2 & s-1 \end{bmatrix} \begin{bmatrix} 1 + \frac{1}{s-2} \\ -2 - \frac{2}{s^2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{s^4 - 6s^3 + 9s^2 - 4s + 8}{s^2(s+1)(s-2)(s-3)} \\ \frac{-2s^4 + 8s^3 - 8s^2 + 6s - 4}{s^2(s+1)(s-2)(s-3)} \end{bmatrix} \end{aligned}$$

Using the method of partial fractions we can write

$$Y_1(s) = \frac{4}{3}\frac{1}{s^2} - \frac{8}{9}\frac{1}{s} + \frac{7}{3}\frac{1}{s+1} - \frac{1}{3}\frac{1}{s-2} - \frac{1}{9}\frac{1}{s-3}$$

$$Y_2(s) = -\frac{2}{3}\frac{1}{s^2} + \frac{10}{9}\frac{1}{s} - \frac{7}{3}\frac{1}{s+1} - \frac{2}{3}\frac{1}{s-2} - \frac{1}{9}\frac{1}{s-3}$$

Therefore

$$y_1(t) = \mathcal{L}^{-1}[Y_1(s)] = \frac{4}{3}t - \frac{8}{9} + \frac{7}{3}e^{-t} - \frac{1}{3}e^{2t} - \frac{1}{9}e^{3t}$$

$$y_2(t) = \mathcal{L}^{-1}[Y_2(s)] = -\frac{2}{3}t + \frac{10}{9} - \frac{7}{3}e^{-t} - \frac{2}{3}e^{2t} - \frac{1}{9}e^{3t}, \quad t \geq 0$$

Hence, for $t \geq 0$

$$\mathbf{y}(t) = t \begin{bmatrix} \frac{4}{3} \\ \frac{2}{3} \\ -3 \end{bmatrix} + \begin{bmatrix} -\frac{8}{9} \\ \frac{10}{9} \\ 9 \end{bmatrix} + e^{7t} \begin{bmatrix} \frac{7}{3} \\ 3 \\ 3 \end{bmatrix} + e^{2t} \begin{bmatrix} -1 \\ \frac{2}{3} \\ -3 \end{bmatrix} + e^{-3t} \begin{bmatrix} -\frac{1}{9} \\ \frac{1}{9} \\ -9 \end{bmatrix} \quad \blacksquare$$

System Transfer Matrix and the Laplace Transform of e^{tA}

The vector equation (8) is a linear time invariant system whose Laplace input is given by $\mathbf{y}_0 + G(s)$ and the Laplace output $\mathbf{Y}(s)$. According to (9) the system tranform matrix is given by $(s\mathbf{I} - \mathbf{A})^{-1}$. We will show that this matrix is the Laplace transform of the exponential matrix function $e^{t\mathbf{A}}$. Indeed, $e^{t\mathbf{A}}$ is the solution to the initial value problem

$$\mathbf{\Phi}'(t) = \mathbf{A}\mathbf{\Phi}(t), \quad \mathbf{\Phi}(0) = \mathbf{I},$$

where $\mathbf{I}$ is the $n \times n$ identity matrix and $\mathbf{A}$ is a constant $n \times n$ matrix. Taking Laplace of both sides yields

$$s\mathcal{L}[\Phi(t)] - \mathbf{I} = \mathbf{A}\mathcal{L}[\Phi(t)].$$

Solving for $\mathcal{L}[\Phi(t)]$ we find

$$\mathcal{L}[\Phi(t)] = (s\mathbf{I} - \mathbf{A})^{-1} = \mathcal{L}[e^{t\mathbf{A}}].$$

Practice Problems

Problem 49.1
Find $\mathcal{L}[\mathbf{y}(t)]$ where

$$\mathbf{y}(t) = \frac{d}{dt}\begin{bmatrix} e^{-t}\cos 2t \\ 0 \\ t + e^{t} \end{bmatrix}$$

Problem 49.2
Find $\mathcal{L}[\mathbf{y}(t)]$ where

$$\mathbf{y}(t) = \int_0^t \begin{bmatrix} 1 \\ u \\ e^{-u} \end{bmatrix} du$$

Problem 49.3
Find $\mathcal{L}^{-1}[\mathbf{Y}(s)]$ where

$$\mathbf{Y}(s) = \begin{bmatrix} \frac{1}{s} \\ \frac{2}{s^2+2s+2} \\ \frac{1}{s^2+s} \end{bmatrix}$$

Problem 49.4
Find $\mathcal{L}^{-1}[\mathbf{Y}(s)]$ where

$$\mathbf{Y}(s) = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} \mathcal{L}[t^3] \\ \mathcal{L}[e^{2t}] \\ \mathcal{L}[\sin t] \end{bmatrix}$$

Problem 49.5
Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 5 & -4 \\ 5 & -4 \end{bmatrix} \mathbf{y} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Problem 49.6

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix} \mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Problem 49.7

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}\mathbf{y} + \begin{bmatrix} 0 \\ 3e^t \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Problem 49.8

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}'' = \begin{bmatrix} -3 & -2 \\ & \end{bmatrix}\mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 1 \\ \end{bmatrix}, \quad \mathbf{y}'(0) = \begin{bmatrix} 0 \\ \end{bmatrix}$$

Problem 49.9

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}'' = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\mathbf{y} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{y}'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Problem 49.10

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & & \end{bmatrix}\mathbf{y} + \begin{bmatrix} e^t \\ 1 \\ -2t \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 0 \\ 0 \\ \end{bmatrix}$$

Problem 49.11

The Laplace transform was applied to the initial value problem $\mathbf{y}' = \mathbf{A}\mathbf{y}$, $\mathbf{y}(0) = \mathbf{y}_0$, where $\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$, $\mathbf{A}$ is a 2×2 constant matrix, and $\mathbf{y}_0 = \begin{bmatrix} y_{1,0} \\ y_{2,0} \end{bmatrix}$.

The following transform domain solution was obtained

$$\mathcal{L}[\mathbf{y}(t)] = \mathbf{Y}(s) = \frac{1}{s^2 - 9s + 18}\begin{bmatrix} s-2 & -1 \\ 4 & s-7 \end{bmatrix}\begin{bmatrix} y_{1,0} \\ y_{2,0} \end{bmatrix}.$$

(a) what are the eigenvalues of $\mathbf{A}$?

(b) Find $\mathbf{A}$.

$y2,0$

Section 43

Problem 43.1

Determine whether the integral $\int_0^\infty \frac{1}{1+t^2}dt$ converges. If the integral converges, give

Solution.

We have

$$\int_0^\infty \frac{1}{1+t^2}dt = \lim_{A\to\infty}\int_0^A \frac{1}{1+t^2}dt = \lim_{A\to\infty}[\arctan t]_0^A$$

$$= \lim_{A\to\infty}\arctan A = \frac{\pi}{2}$$

So the integral is convergent ■

Problem 43.2

Determine whether the integral $\int_0^\infty \frac{t}{1+t^2}dt$ converges. If the integral converges, give

Solution.

We have

$$\int_0^\infty \frac{t}{1+t^2}dt = \frac{1}{2}\lim_{A\to\infty}\int_0^A \frac{2t}{1+t^2}dt = \frac{1}{2}\lim_{A\to\infty}\left[\ln(1+t^2)\right]_0^A$$

$$= \frac{1}{2}\lim_{A\to\infty}\ln(1+A^2) = \infty$$

Hence, the integral is divergent ■

Problem 43.3

Determine whether the integral $\int_0^\infty e^{-t}\cos(e^{-t})dt$ converges. If the integral converges, give its value.

Solution.

Using the substitution $u = e^{-t}$ we find

$$\int_0^\infty e^{-t}\cos(e^{-t})dt = \lim_{A\to\infty}\int_{e^{-A}}^{1}\cos u\,du$$

$$= \lim_{A\to\infty}[-\sin u]_1^{e^{-A}} = \lim_{A\to\infty}[\sin 1 - \sin(e^{-A})]$$

$$= \sin 1$$

Hence, the integral is convergent ■

Problem 43.4

Using the definition, find $L[e^{3t}]$, if it exists. If the Laplace transform existsthen find the domain of $F(s)$.

Solution.

We have

$$L[e^{3t}] = \lim_{A\to\infty}\int_0^A e^{3t}e^{-st}dt = \lim_{A\to\infty}\int_0^A e^{t(3-s)}dt$$

$$= \lim_{A\to\infty}\left[\frac{e^{t(3-s)}}{3-s}\right]_0^A$$

$$= \lim_{A\to\infty}\left(\frac{e^{A(3-s)}}{3-s} - \frac{1}{3-s}\right)$$

$$= \frac{1}{s-3},\quad s > 3$$ ■

Problem 43.5

Using the definition, find $[t \quad 5]$, if it exists. If the Laplace transform existsthen find the domain of $F(s)$.

Solution.

Using integration by parts we find

$$L[t-5] = \lim_{A\to\infty}\int_0^A (t-5)e^{-st}dt = \lim_{A\to\infty}\left(\frac{-(t-5)e^{-st}}{s}\Big|_0^A \quad \frac{1}{s}\int_0^A e^{-st}dt\right)$$

$$= \lim_{A\to\infty}\left(\frac{-(A-5)e^{-sA}+5}{s} \quad \frac{e^{-st}}{s^2}\Big|_0^A\right)$$

$$= \frac{1}{s^2} - \frac{5}{s}, \quad s > 0 \blacksquare$$

Problem 43.6

Using the definition, find $\mathcal{L}[e^{(t-1)^2}]$, if it exists. If the Laplace transformexists then find the domain of $F(s)$.

Solution.

We have

$$\int_0^\infty e^{(t-1)^2}e^{-st}dt = \int_0^\infty e^{(t-1)^2-st}dt.$$

Since $\lim_{t\to\infty}(t-1)^2 - st = \lim_{t\to\infty} t^2\left(1 - \frac{(2+s)}{t} + \frac{1}{t^2}\right) = \infty$, for any fixed s we can choose a positive C such that $(t-1)^2 - st \geq 0$ for $t \geq C$. In this case, $e^{(t-1)^2-st} \geq 1$ and this implies that $\int_0^\infty e^{(t-1)^2-st}dt \geq \int_C^\infty dt$. The integral on the right is divergent so that the integral on the left is also divergent by the comparison theorem of improper integrals. Hence, $f(t) = e^{(t-1)^2}$ does not have a Laplace transform ■

Problem 43.7

Using the definition, find $\mathcal{L}[(t-2)^2]$, if it exists. If the Laplace transformexists then find the domain of $F(s)$.

Solution.

We have

$$\mathcal{L}[(t-2)^2] = \lim_{T\to\infty}\int_0^T (t-2)^2e^{-st}dt.$$

Using integration by parts with $u' = e^{-st}$ and $v = (t-2)^2$ we find

$$\int_0^T (t-2)^2e^{-st}dt = -\left[\frac{(t-2)^2e^{-st}}{s}\right]_0^T + \frac{2}{s}\int_0^T (t-2)e^{-st}dt$$

$$= \frac{4}{s} - \frac{(T-2)^2e^{-sT}}{s} + \frac{2}{s}\int_0^T (t-2)e$$

$^{-st}dt.$

Thus,

$$\lim_{T\to\infty}\int_0^T (t-2)^2 e^{-st}dt = \frac{4}{s} + \frac{2}{s}\lim_{T\to\infty}\int_0^T (t-2)e^{-st}dt.$$

Using by parts with $u' = e^{-st}$ and $v = t-2$ we find

$$\int_0^T (t-2)e^{-st}dt = \left[-\frac{(t-2)e^{-st}}{s} - \frac{1}{s^2}e^{-st}\right]_0^T.$$

Letting $T \to \infty$ in the above expression we find

$$\lim_{T\to\infty}\int_0^T (t-2)e^{-st}dt = \frac{}{s} + \frac{1}{s^2}, \quad s > 0.$$

Hence,

$$F(s) = \frac{4}{s} + \frac{2}{s} - \frac{2}{s} + \frac{1}{s^2} = \frac{4}{s} - \frac{4}{s^2} + \frac{2}{s^3}, \quad s > 0 \blacksquare$$

Problem 43.8

Using the definition, find $\mathcal{L}[f(t)]$, if it exists. If the Laplace transform existsthen find the domain of $F(s)$.

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t-1, & t \ge 1 \end{cases}$$

Solution.

We have

$$\mathcal{L}[f(t)] = \lim_{T\to\infty}\int_1^T (t-1)e^{-st}dt.$$

Using integration by parts with $u' = e^{-st}$ and $v = t-1$ we find

$$\lim_{T\to\infty}\int_1^T (t-1)e^{-st}dt = \lim_{T\to\infty}\left[-\frac{(t-1)e^{-st}}{s} - \frac{1}{s^2}e^{-st}\right]_1^T = \frac{e^{-s}}{s^2}, \quad s > 0 \blacksquare$$

Problem 43.9

Using the definition, find $\mathcal{L}[f(t)]$, if it exists. If the Laplace transform existsthen find the domain of $F(s)$.

Solution.

We have

$$f(t)=\begin{cases}0, & 0\le t<1\\ t-1, & 1\le t<2\\ 0, & t\ge 2.\end{cases}$$

$$L[f(t)] = \int_1^2 (t-1)e^{-st}dt = -\frac{(t-1)e^{-st}}{s}\Big|_1^2 - \frac{1}{s}e^{-st}\Big|_1^2$$

e^{-2s} $\frac{1}{s}$ $-$ $-$

$-2s$

$= - \quad s \quad + \quad s^2 (e \quad -e \qquad), \quad s \neq 0$ ■

Problem 43.10

Let n be a positive integer. Using integration by parts establish the reduction formula

$$\int t^n e^{-st}\,dt = -\frac{t^n e^{-st}}{s} + \frac{n}{s}\int t^{n-1} e^{-st}\,dt, \quad s > 0.$$

Solution.

Let $u' = e^{-st}$ and $v = t^n$. Then $u = -\frac{e^{-st}}{s}$ and $v' = nt^{n-1}$. Hence,

$$\int t^n e^{-st}\,dt = -\frac{t^n e^{-st}}{s} + \frac{n}{s}\int t^{n-1} e^{-st}\,dt, \quad s > 0 \ \blacksquare$$

Problem 43.11

For $s > 0$ and n a positive integer evaluate the limits

(a) $\lim_{t\to 0} t^n e^{-st}$ (b) $\lim_{t\to\infty} t^n e^{-st}$

Solution.

(a) $\lim_{t\to 0} t^n e^{-st} = \lim_{t\to 0} \frac{t^n}{e^{st}} = \frac{0}{1} = 0.$

(b) Using L'Hôpital's rule repeatedly we find

$$\lim_{t\to\infty} t^n e^{-st} = \cdots = \lim_{t\to\infty} \frac{n!}{s^n e^{st}} = 0 \ \blacksquare$$

Problem 43.12

(a) Use the previous two problems to derive the reduction formula for the Laplace transform of $f(t) = t^n$,

$$\mathcal{L}[t^n] = \frac{n}{s}\mathcal{L}[t^{n-1}], \quad s > 0.$$

(b) Calculate $[t^k]$, for $k = 1, 2, 3, 4, 5.$

(c) Formulate a conjecture as to the Laplace transform of $f(t)$, t^n with n a positive integer.

Solution.

(a) Using the two previous problems we find

$$\mathcal{L}[t^n] = \lim_{T\to\infty}\int_0^T t^n e^{-st}dt = \lim_{T\to\infty}\left(-\frac{t^n e^{-st}}{s}\Big|_0^T + \frac{n}{s}\int_0^T t^{n-1}e^{-st}dt\right)$$

$$= \frac{n}{s}\lim_{T\to\infty}\int_0^T t^{n-1}e^{-st}dt = \frac{n}{s}\mathcal{L}[t^{n-1}], \quad s > 0$$

(b) We have

$$L[t] = \frac{1}{s^2}$$

$$L[t^2] = \frac{2}{s}L[t] = \frac{2}{s^3}$$

$$L[t^3] = \frac{3}{s}L[t^2] = \frac{6}{s^4}$$

$$L[t^4] = \frac{4}{s}L[t^3] = \frac{24}{s^5}$$

$$L[t^5] = \frac{5}{s}L[t^4] = \frac{120}{s^5}$$

(c) By induction, one can easily show that for $n = 0, 1, 2, \cdots$

$$L[t^n] = \frac{n!}{s^{n+1}}, \quad s > 0 \blacksquare$$

Problem 43.13

From a table of integrals,

$$\int e^{\alpha u} \sin \beta u du = e^{\alpha u} \frac{\alpha \sin \beta u - \beta \cos \beta u}{\alpha^2 + \beta^2}$$

$$\int e^{\alpha u} \cos \beta u du = e^{\alpha u} \frac{\alpha \cos \beta u + \beta \sin \beta u}{\alpha^2 + \beta^2}$$

Use the above integrals to find the Laplace transform of $f(t) = \cos \omega t$, if it exists. If the Laplace transform exists, give the domain of $F(s)$.

Solution.

We have

$$L[\cos \omega t] = \lim_{T \to \infty} \left(e^{-st} \frac{-s \cos \omega t + \omega \sin \omega t}{s^2 + \omega^2} \Big|_0^T \right) = \frac{s}{s^2 + \omega^2}, \quad s > 0 \blacksquare$$

$$\lim_{T\to\infty} \quad \frac{}{s^2+\omega^2} \quad 0 \quad s^2+\omega^2$$

Problem 43.14

Use the above integrals to find the Laplace transform of $f(t) = \sin \omega t$, if it exists. If the Laplace transform exists, give the domain of $F(s)$.

Solution.

We have

$$\mathcal{L}[\sin \omega t] = \lim_{T\to\infty} \left(-e^{-st} \frac{-s \sin \omega t + \omega \cos \omega t}{s^2+\omega^2} \right)\Bigg|_0^T = \frac{\omega}{s^2+\omega^2}, \quad s > 0 \blacksquare$$

Problem 43.15

Use the above integrals to find the Laplace transform of $f(t) = \cos\omega(t - 2)$, if it exists. If the Laplace transform exists, give the domain of $F(s)$.

Solution.

Using a trigonometric identity we can write $f(t) = \cos\omega(t - 2) = \cos\omega t \cos 2\omega + \sin\omega t \sin 2\omega$. Thus, using the previous two problems we find

$$\mathcal{L}[\cos\omega(t-2)] = \frac{s\cos 2\omega + \omega\sin 2\omega}{s^2+\omega^2}, \quad s > 0 \blacksquare$$

Problem 43.16

Use the above integrals to find the Laplace transform of $f(t) = e^{3t}\sin t$, if it exists. If the Laplace transform exists, give the domain of $F(s)$.

Solution.

We have

$$\begin{aligned}
\mathcal{L}[e^{3t}\sin t] &= \lim_{T\to\infty}\int_0^T e^{-(s-3)t}\sin t\,dt \\
&= \lim_{T\to\infty}\left(-e^{-(s-3)t}\frac{(s-3)\sin t+\cos t}{(s-3)^2+1}\Big|_0^T\right) \\
&= \frac{1}{(s-3)^2+1}, \quad s > 3 \blacksquare
\end{aligned}$$

Problem 43.17

Use the linearity property of Laplace transform to find $\mathcal{L}[5e^{-7t} + t + 2e^{2t}]$. Find the domain of $F(s)$.

Solution.

We have $\mathcal{L}[e^{-7t}] = \frac{1}{s+7}$, $s > -7$, $\mathcal{L}[t] = \frac{1}{s^2}$, $s > 0$, and $\mathcal{L}[e^{2t}] = \frac{1}{s-2}$, $s > 2$. Hence,

$$\mathcal{L}[5e^{-7t} + t + 2e^{2t}] = 5\mathcal{L}[e^{-7t}] + \mathcal{L}[t] + 2\mathcal{L}[e^{2t}] = \frac{5}{s+7} + \frac{1}{s^2} + \frac{2}{s-2}, \quad s > 2 \blacksquare$$

Problem 43.18

Consider the function $f(t) = \tan t$.

(a) Is $f(t)$ continuous on $0 \le t < \infty$, discontinuous but piecewise continuous on $0 \le t < \infty$, or neither?

(b) Are there fixed numbers a and M such that $|f(t)| \le Me^{at}$ for $0 \le t < \infty$?

Solution.

(a) Since $f(t) = \tan t = \frac{\sin t}{\cos t}$ and this function is discontinuous at $t = (2n + 1)\frac{\pi}{2}$. Since this function has vertical asymptotes there it is not piecewise continuous.

(b) The graph of the function does not show that it can be bounded by exponential functions. Hence, no such numbers a and M ■

Problem 43.19

Consider the function $f(t) = t^2e^{-t}$.

(a) Is $f(t)$ continuous on $0 \le t < \infty$, discontinuous but piecewise continuous on $0 \le t < \infty$, or neither?

(b) Are there fixed numbers a and M such that $|f(t)| \le Me^{at}$ for $0 \le t < \infty$?

Solution.

(a) Since t^2 and e^{-t} are continuous everywhere, $f(t) = t^2e^{-t}$ is continuous on $0 \le t < \infty$.

(b) By L'Hôpital's rule one has

$$\lim_{t\to\infty} \frac{t^2}{e^{t}} = 0$$

Since $f(0) = 0$, $f(t)$ is bounded. Since $f'(t) = (2t - t^2)e^{-t}$, $f(t)$ has a maximum when $t = 2$. The value of this maximum is $f(2) = 4e^{-2}$. Hence, $M = 4e^{-2}$ and $a = 0$ ■

Problem 43.20

Consider the function $f(t) = \frac{e^{t^2}}{e^{2t}+1}$.

(a) Is $f(t)$ continuous on $0 \le t < \infty$, discontinuous but piecewise continuous on $0 \le t < \infty$, or neither?

(b) Are there fixed numbers a and M such that $|f(t)| \le Me^{at}$ for $0 \le t < \infty$?

Solution.

(a) Since e^{t^2} and $e^{2t}+1$ are continuous everywhere, $f(t) = \frac{e^{t^2}}{e^{2t}+1}$ is continuous on $0 \le t < \infty$.

(b) Since $e^{2t} + 1 \le e^{2t} + e^{2t} = 2e^{2t}$, $f(t) \ge \frac{1}{2} e^{t^2} e^{-2t} = \frac{1}{2} e^{t^2 - 2t}$. But for $t \ge 4$ we have $t^2 - 2t > \frac{t^2}{2}$. Hence, $f(t) > \frac{1}{2} e^{\frac{t^2}{2}}$. So $f(t)$ is not of exponential order at infinity ■

Problem 43.21

Consider the floor function $f(t) = \lfloor t \rfloor$, where for any integer n we have $\lfloor t \rfloor = n$ for all $n \leq t < n+1$.

(a) Is $f(t)$ continuous on $0 \leq t < \infty$, discontinuous but piecewise continuous on $0 \leq t < \infty$, or neither?
(b) Are there fixed numbers a and M such that $|f(t)| \leq Me^{at}$ for $0 \leq t < \infty$?

Solution.
(a) The floor function is a piecewise continuous function on $0 \leq t < \infty$.
(b) Since $\lfloor t \rfloor \leq t < e^t$ for $0 \geq t < \infty$ we find $M = 1$ and $a = 1$ ■

Problem 43.22

Find $\mathcal{L}^{-1}\left[\frac{3}{s-2}\right]$.

Solution.
Since $\mathcal{L}\left[e^{at}\right] = \frac{1}{s-a}$, $s > a$ we find

$$\mathcal{L}^{-1}\left[\frac{3}{s-2}\right] = 3\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = 3e^{2t}, \quad t \geq 0$$ ■

Problem 43.23

Find $\mathcal{L}^{-1}\left[-\frac{2}{s^2} + \frac{1}{s+1}\right]$.

Solution.
Since $\mathcal{L}[t] = \frac{1}{s^2}$, $s > 0$ and $\mathcal{L}\left[e^{at}\right] = \frac{1}{s-a}$, $s > a$ we find

$$\mathcal{L}^{-1}\left[-\frac{2}{s^2} + \frac{1}{s+1}\right] = -2\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] + \mathcal{L}^{-1}\left[\frac{1}{s+1}\right]$$

$$= -2t + e^{-t}, \ t \geq 0$$ ■

Problem 43.24

Find $\mathcal{L}^{-1}\left[\frac{2}{s+2} + \frac{2}{s-2}\right]$.

Solution.
We have

$$L^{-1}\left\{\frac{2}{s+2}+\frac{2}{s-2}\right\}=2L^{-1}\left\{\frac{1}{s+2}\right\}+2L^{-1}\left\{\frac{1}{s-2}\right\}=2(e^{-2t}+e^{2t}),\ t\geq 0 \quad \blacksquare$$

Section 44

Problem 44.1

Use Table L to find $L[2e^t + 5]$.

Solution.

$$L[2e^t + 5] = 2L[e^t] + 5L[1] = \frac{2}{s-1} + \frac{5}{s}, \quad s > 1 \blacksquare$$

Problem 44.2

Use Table L to find $L[e^{3t-3}h(t-1)]$.

Solution.

$$L[e^{3t-3}h(t-1)] = L[e^{3(t-1)}h(t-1)] = e^{-s}L[e^{3t}] = \frac{e^{-s}}{s-3}, \quad s > 3 \blacksquare$$

Problem 44.3

Use Table L to find $L[\sin^2 \omega t]$.

Solution.

$$L[\sin^2 \omega t] = L\left[\frac{1-\cos 2\omega t}{2}\right] = \frac{1}{2}(L[1]-L[\cos 2\omega t]) = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{s}{s^2+4\omega^2}, \quad s > 0 \blacksquare$$

Problem 44.4

Use Table L to find $L[\sin 3t \cos 3t]$.

Solution.

$$L[\sin 3t \cos 3t] = L\left[\frac{\sin 6t}{2}\right] = \frac{1}{2}L[\sin 6t] = \frac{3}{s^2+36}, \quad s > 0 \blacksquare$$

Problem 44.5

Use Table L to find L$[e^{2t}\cos 3t]$.

Solution.

$$[e^{2t}\cos 3t] = \frac{s-2}{(s-2)^2+9}, \quad s > 2 \blacksquare$$

Problem 44.6

Use Table L to find $\mathcal{L}[e^{4t}(t^2+3t+5)]$.

Solution.

$$\mathcal{L}[e^{4t}(t^2+3t+5)] = \mathcal{L}[e^{4t}t^2]+3\mathcal{L}[e^{4t}t]+5\mathcal{L}[e^{4t}] = \frac{2}{(s-4)^3}+\frac{3}{(s-4)^2}+\frac{5}{s-4}, \quad s>4 \blacksquare$$

Problem 44.7

Use Table L to find $\mathcal{L}^{-1}[\frac{10}{s^2+25}+\frac{4}{s-3}]$.

Solution.

$$\mathcal{L}^{-1}\left[\frac{10}{s^2+25}+\frac{4}{s-3}\right] = 2\mathcal{L}^{-1}\left[\frac{5}{s^2+25}\right]+4\mathcal{L}^{-1}\left[\frac{1}{s-3}\right] = 2\sin 5t+4e^{3t}, \quad t\ge 0 \blacksquare$$

Problem 44.8

Use Table to find $\mathcal{L}^{-1}[\frac{5}{(s-3)^4}]$.

Solution.

$$\mathcal{L}^{-1}\left[\frac{5}{(s-3)^4}\right] = \frac{5}{6}\mathcal{L}^{-1}\left[\frac{3!}{(s-3)^4}\right] = \frac{5}{6}e^{3t}t^3, \quad t\ge 0 \blacksquare$$

Problem 44.9

Use Table to find $\mathcal{L}^{-1}[\frac{e^{-2s}}{s-9}]$.

Solution.

$$\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s-9}\right] = e^{9(t-2)}h(t-2) = \begin{cases} 0, & 0\le t<2 \\ e^{9(t-2)}, & t\ge 2 \end{cases} \blacksquare$$

Problem 44.10

Use Table L to find $\mathcal{L}^{-1}[\frac{e^{-3s}(2s+7)}{s^2+16}]$.

Solution.

We have

$$\mathcal{L}^{-1}\left[\frac{e^{-3s}(2s+7)}{s^2+16}\right] = 2\mathcal{L}^{-1}\left[\frac{s\,e^{-3s}}{s^2+16}\right] + \frac{7}{4}\mathcal{L}^{-1}\left[\frac{4e^{-3s}}{s^2+16}\right]$$

$$= 2\cos 4(t-3)h(t-3) + \frac{7}{4}\sin 4(t-3)h(t-3), \quad t \geq 0 \qquad \blacksquare$$

Problem 44.11

Graph the function $f(t) = h(t-1) + h(t-3)$ for $t \geq 0$, where $h(t)$ is the Heaviside step function, and use Table L to find $\mathcal{L}[f(t)]$.

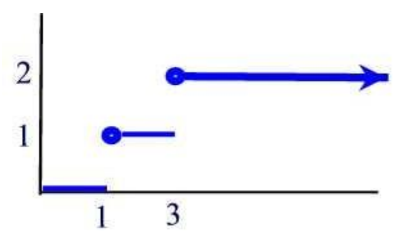
Solution.

Note that

$$f(t) = \begin{cases} 0, & 0 \quad t < 1 \\ 1, & 1 \quad t < 3 \\ 2, & t \geq 3 \end{cases}$$

The graph of $f(t)$ is shown below. Using Table L we find

$$\mathcal{L}[f(t)] = \mathcal{L}[h(t-1)] + \mathcal{L}[h(t-3)] = \frac{e^{-s}}{s} + \frac{e^{-3s}}{s}, \quad s > 0 \blacksquare$$



Problem 44.12

Graph the function $f(t) = t[h(t-1) - h(t-3)]$ for $t \geq 0$, where $h(t)$ is the Heaviside step function, and use Table L to find $\mathcal{L}[f(t)]$.

Solution.

Note that

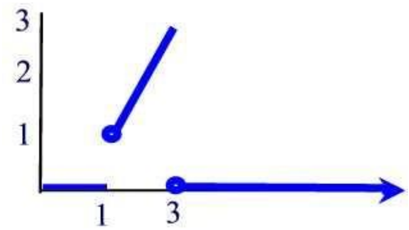
$$f(t) = \begin{cases} 0, & 0 \quad t < 1 \\ t, & 1 \quad t < 3 \\ 0, & t \geq 3 \end{cases}$$

The graph of $f(t)$ is shown below. Using Table L we find

$$\mathcal{L}[f(t)] = \mathcal{L}[(t-1)h(t-1) + h(t-1) - (t-3)h(t-3) - 3h(t-3)]$$

$$= \mathcal{L}[(t-1)h(t-1)] + \mathcal{L}[h(t-1)] - \mathcal{L}[(t-3)h(t-3)] - 3\mathcal{L}[h(t-3)]$$

$$= \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s} - \frac{e^{-3s}}{s^2} - \frac{3e^{-3s}}{s}, \quad s > 1 \blacksquare$$



Problem 44.13

Graph the function $f(t) = 3[h(t-1) - h(t-4)]$ for $t \geq 0$, where $h(t)$ is the Heaviside step function, and use Table L to find $\mathcal{L}[f(t)]$.

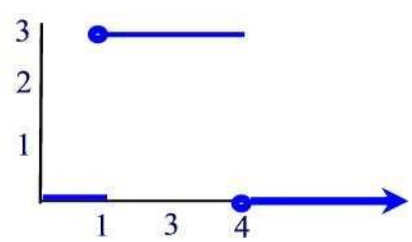
Solution.

Note that

$$f(t) = \begin{cases} 0, & 0 \quad t < 1 \\ 3, & 1 \quad t < 4 \\ 0, & t \geq 4 \end{cases}$$

The graph of $f(t)$ is shown below. Using Table L we find

$$\mathcal{L}[f(t)] = 3\mathcal{L}[h(t-1)] - 3\mathcal{L}[h(t-4)] = \frac{3e^{-s}}{s} - \frac{3e^{-4s}}{s}, \quad s > 0 \blacksquare$$



Problem 44.14

Graph the function $f(t) = |2-t|[h(t-1) - h(t-3)]$ for $t \geq 0$, where $h(t)$ is the Heaviside step function, and use Table L to find $\mathcal{L}[f(t)]$.

Solution.

Note that

$$f(t) =$$

$0, \; 0 \leq t < 1$

$|2 - t|, \; 1 \leq t < 3$

$0, \; t \geq 3$

The graph of $f(t)$ is shown below. Using Table L we find

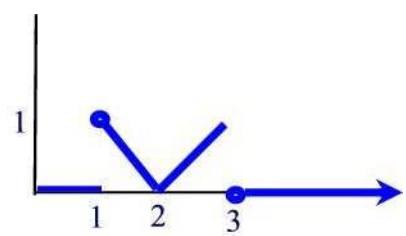
$$L[f(t)] = (2-t)h(t-1) + 2(t-2)h(t-2) - (t-2)h(t-3)$$

$$= L[-(t-1)h(t-1) + h(t-1) + 2(t-2)h(t-2) - (t-3)h(t-3) - h(t-3)]$$

$$= -L[(t-1)h(t-1)] + L[h(t-1)] + 2L[(t-2)h(t-2)]$$

$$-L[(t-3)h(t-3)] - L[h(t-3)]$$

$$= -\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s} + \frac{2e^{-2s}}{s^2} - \frac{e^{-3s}}{s^2} - \frac{e^{-3s}}{s}, \quad s > 0 \blacksquare$$



Problem 44.15

Graph the function $f(t) = h(2-t)$ for $t \geq 0$, where $h(t)$ is the Heaviside step function, and use Table L to find $L[f(t)]$.

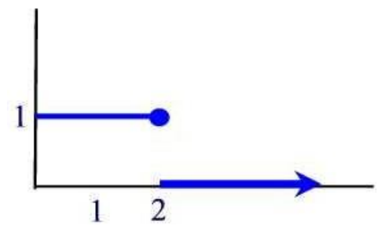
Solution.

Note that

$$f(t) = \begin{cases} 1, & 0 \leq t \leq 2 \\ 0, & t > 2 \end{cases}$$

The graph of $f(t)$ is shown below. From this graph we see that $f(t) = h(t) - h(t-2)h(t-2)$. Using Table L we find

$$L[f(t)] = L[h(t)] - L[h(t-1)h(t-1)] = \frac{1 - e^{-2s}}{s}, \quad s > 0 \blacksquare$$


1
1
2

Problem 44.16

Graph the function $f(t) = h(t-1) + h(4-t)$ for $t \geq 0$, where $h(t)$ is the Heaviside step function, and use Table L to find $\mathcal{L}[f(t)]$.

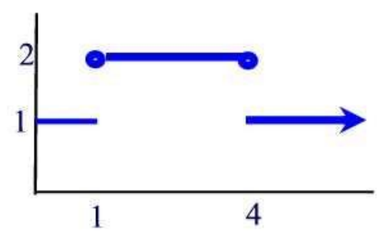
Solution.

Note that

$$f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t \leq 4 \\ 1, & t > 4 \end{cases}$$

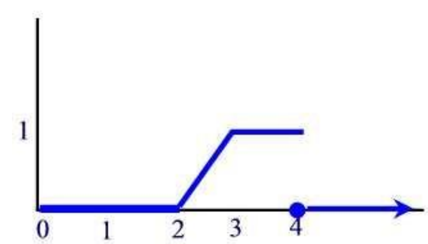
The graph of $f(t)$ is shown below. Using Table L we find

$$\mathcal{L}[f(t)] = \mathcal{L}[h(t-1)] + \mathcal{L}[h(4-t)] = \frac{e^{-s}}{s} + \int_0^4 e^{-st}\,dt = \frac{1 + e^{-s} - e^{-4s}}{s}, \; s > 0 \blacksquare$$



Problem 44.17

The graph of $f(t)$ is given below. Represent $f(t)$ as a combination of Heaviside step functions, and use Table to calculate the Laplace transform of $f(t)$.



Solution.

From the graph we see that

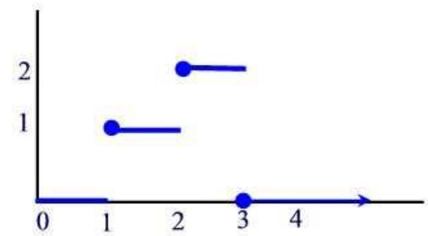
$$f(t) = (t-2)h(t-2) - (t-3)h(t-3) - h(t-4)$$

Thus,

$$L[f(t)] = L[(t-2)h(t-2)] - L[(t-3)h(t-3)] - L[h(t-4)] = \frac{e^{-2s}}{s^2} - \frac{e^{-3s}}{} - \frac{e^{-4s}}{s},\ s > 0 \blacksquare$$

Problem 44.18

The graph of $f(t)$ is given below. Represent $f(t)$ as a combination of Heaviside step functions, and use Table to calculate the Laplace transform of $f(t)$.



Solution.

From the graph we see that

$$f(t) = h(t-1) + h(t-2) - 2h(t-3).$$

Thus,

$$\mathcal{L}[f(t)] = \mathcal{L}[h(t-1)] - 2\mathcal{L}[h(t-3)] + \mathcal{L}[h(t-2)] = \frac{e^{-s} - 2e^{-3s} + e^{-2s}}{s}, \quad s > 0 \blacksquare$$

Problem 44.19

Using the partial fraction decomposition find $\mathcal{L}^{-1}\left[\frac{12}{(s-3)(s+1)}\right]$

Solution.

Write

$$\frac{12}{(s-3)(s+1)} = \frac{A}{s-3} + \frac{B}{s+1}.$$

Multiply both sides of this equation by $s-3$ and cancel common factors to obtain

$$\frac{12}{s+1} = A + \frac{B(s-3)}{s+1}.$$

Now, find A by setting $s = 3$ to obtain $A = 3$. Similarly, by multiplying both sides by $s + 1$ and then setting $s = -1$ in the resulting equation leads to $B = -3$. Hence,

$$\frac{12}{(s-3)(s+1)} = 3\left(\frac{1}{s-3} - \frac{1}{s+1}\right).$$

Finally,

$$\mathcal{L}^{-1}\left[\frac{12}{(s-3)(s+1)}\right]=3\mathcal{L}^{-1}\left[\frac{1}{s-3}\right]-3\mathcal{L}^{-1}\left[\frac{1}{s+1}\right]$$

$$=3e^{3t}-3e^{-t},\ t\geq 0 \quad \blacksquare$$

Problem 44.20
Using the partial fraction decomposition find $\mathcal{L}^{-1}\left[\frac{24e^{-5s}}{s^2-9}\right]$.

Solution.
Write

$$\frac{24}{(s-3)(s+3)}=\frac{A}{s-3}+\frac{B}{s+3}.$$

Multiply both sides of this equation by $s-3$ and cancel common factors to obtain

$$\frac{24}{s+3}=A+\frac{B(s-3)}{s+3}.$$

Now, find A by setting $s=3$ to obtain $A=4$. Similarly, by multiplying both sides by $s+3$ and then setting $s=-3$ in the resulting equation leads to $B=-4$. Hence,

$$\frac{24}{(s-3)(s+3)}=4\left[\frac{1}{s-3}-\frac{1}{s+3}\right].$$

Finally,

$$\mathcal{L}^{-1}\left[\frac{24e^{-5s}}{(s-3)(s+3)}\right]=4\mathcal{L}^{-1}\left[\frac{e^{-5s}}{s-3}\right]-4\mathcal{L}^{-1}\left[\frac{e^{-5s}}{s+3}\right]$$

$\blacksquare$

Problem 44.21 $=4[e^{3(t-5)} - e^{-3(t-5)}]h(t-5), \quad t \geq 0$

Use Laplace transform technique to solve the initial value problem

$$y' + 4y = g(t), \quad y(0) = 2$$

where

$$g(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 12, & 1 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

Solution.

Note first that $g(t) = 12[h(t-1) - h(t-3)]$ so that

$$\mathrm{L}[g(t)] = 12\mathrm{L}[h(t-1)] - 12\mathrm{L}[h(t-3)] = \frac{12(e^{-s} - e^{-3s})}{s}, \quad s > 0.$$

Now taking the Laplace transform of the DE and using linearity we find

$$\mathbf{L}[y'] + 4\mathbf{L}[y] = \mathbf{L}[g(t)].$$

But $\mathbf{L}[y'] = s\mathbf{L}[y] - y(0) = s\mathbf{L}[y] - 2$. Letting $\mathbf{L}[y] = Y(s)$ we obtain

$$sY(s) - 2 + 4Y(s) = 12\frac{e^{-s} - e^{-3s}}{s}.$$

Solving for $Y(s)$ we find

$$Y(s) = \frac{2}{s+4} + 12\frac{e^{-s} - e^{-3s}}{s(s+4)}.$$

But

$$\mathrm{L}^{-1}\left[\frac{2}{s+4}\right] = 2e^{-4t}$$

and

$$\mathrm{L}^{-1}\left[\frac{e^{-s} - e^{-3s}}{s(s+4)}\right] = 3\mathrm{L}^{-1}\left[(e^{-s} - e^{-3s})\left(\frac{1}{s} - \frac{1}{s+4}\right)\right]$$

$-1 \quad e^{-s}$ $\quad - 3\text{L} \quad {}^{-1} e^{-3} \; s \quad - 3\text{L} \quad {}^{-1} \; e^{-s} \quad + 3\text{L} \quad {}^{-1} \; -3s^{e}$

$\overline{}$ s $\quad \overline{}$ s $\quad \overline{}$ $s + 4$ $\quad \overline{}$ $s + 4$

$$=3h(t-1) - 3h(t-3) - 3e^{-4(t-1)}h(t-1) + 3e^{-4(t-3)}h(t-3)$$

Hence,

$$y(t) = 2e^{-4t}+3[h(t-1)-h(t-3)]-3[e^{-4(t-1)}h(t-1)-e^{-4(t-3)}h(t-3)], \quad t \geq 0 \qquad \blacksquare$$

Problem 44.22

Use Laplace transform technique to solve the initial value problem

$$y'' - 4y = e^{3t}, \quad y(0) = 0, \quad y'(0) = 0.$$

Solution.

Taking the Laplace transform of the DE and using linearity we find

$$\mathbf{L}[y''] - 4\mathbf{L}[y] = \mathbf{L}[e^{3t}].$$

But $\mathbf{L}[y''] = s^2\mathbf{L}[y] - sy(0) - y'(0) = s^2\mathbf{L}[y]$. Letting $\mathbf{L}[y] = Y(s)$ we obtain

$$s^2Y(s) \quad 4Y(s) = \frac{1}{s-3}.$$

Solving for $Y(s)$ we find

$$Y(s) = \frac{1}{(s-3)(s-2)(s+2)}.$$

Using partial fraction decomposition

$$\frac{1}{(s-3)(s-2)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2} + \frac{C}{s-2}$$

we find $A = \frac{1}{5}$, $B = \frac{1}{20}$, and $C = -\frac{1}{4}$. Thus,

$$y(t) = \mathbf{L}^{-1}\left[\frac{1}{(s-3)(s-2)(s+2)}\right] = \frac{1}{5}\mathbf{L}^{-1}\left[\frac{1}{s-3}\right] + \frac{1}{20}\mathbf{L}^{-1}\left[\frac{1}{s+2}\right] - \frac{1}{4}\mathbf{L}^{-1}\left[\frac{1}{s-2}\right]$$

$$= \frac{1}{5}e^{3t} + \frac{1}{20}e^{-2t} - \frac{1}{4}e^{2t}, \quad t \geq 0 \quad \blacksquare$$

Problem 44.23

Obtain the Laplace transform of the function $\int^t f(\lambda)d\lambda$ in terms of $\mathbf{L}[f(t)] = F(s)$ given that

Solution.

We have

$$\int_0^2 f(\lambda)d\lambda = 3.$$

$$\mathcal{L}\left[\int_2^t f(\lambda)d\lambda\right] = \mathcal{L}\left[\int_0^t f(\lambda)d\lambda - \int_0^2 f(\lambda)d\lambda\right]$$

$$= \frac{F(s)}{s} - \mathcal{L}[3]$$

$$= \frac{F(s)}{s} - \frac{3}{s}, \quad s > 0 \ \blacksquare$$

Section 45

In Problems 45.1 - 45.4, give the form of the partial fraction expansion for $F(s)$. You need not evaluate the constants in the expansion. However, if the denominator has an irreducible quadratic expression then use the completing the square process to write it as the sum/difference of two squares.

Problem 45.1

$$F(s) = \frac{s^3 + 3s + 1}{(s-1)^3(s-2)^2}.$$

Solution.

$$F(s) = \frac{A_1}{(s-1)^3} + \frac{A_2}{(s-1)^2} + \frac{A_3}{s-1} + \frac{B_1}{(s-2)^2} + \frac{B_2}{s-2} \blacksquare$$

Problem 45.2

$$F(s) = \frac{s^2 + 5s \quad 3}{(s^2+16)(s-2)}.$$

Solution.

$$F(s) = \frac{A_1 s + A_2}{s^2 + 16} + \frac{B_1}{s-2} \blacksquare$$

Problem 45.3

$$F(s) = \frac{s^3 - 1}{(s^2+1)^2(s+4)^2}.$$

Solution.

$$F(s) = \frac{A_1 s + A_2}{(s^2+1)^2} + \frac{A_3 s + A_4}{s^2+1} + \frac{B_1}{(s+4)^2} + \frac{B_2}{s+4} \blacksquare$$

Problem 45.4

$$F(s) = \frac{s^4 + 5s^2 + 2s \quad 9}{(s^2 + 8s + 17)(s-2)^2}.$$

Solution.

$$F(s) = \frac{A_1}{(s-2)^2} + \frac{A_2}{s-2} + \frac{B_1 s + B_2}{(s+4)^2 + 1} \blacksquare$$

Problem 45.5

$$\frac{1}{(s+1)^3}$$

Solution.

Using Table L we find $\mathcal{L}^{-1}\left(\frac{1}{(s+1)^3}\right) = \frac{1}{2}e^{-t}t^2, \ t \ge 0 \blacksquare$

Problem 45.6

Find $\mathcal{L}^{-1}\left(\frac{2s-3}{s^2-3s+2}\right)$.

Solution.

We factor the denominator and split the rational function into partial fractions:

$$\frac{2s-3}{(s-1)(s-2)} = \frac{A}{s-1} + \frac{B}{s-2}.$$

Multiplying both sides by $(s-1)(s-2)$ and simplifying to obtain

$$\begin{aligned} 2s - 3 &= A(s-2) + B(s-1) \\ &= (A+B)s - 2A - B. \end{aligned}$$

Equating coefficients of like powers of s we obtain the system

$$\begin{aligned} A + B &= 2 \\ -2A - B &= -3. \end{aligned}$$

Solving this system by elimination we find $A = 1$ and $B = 1$. Now findingthe inverse Laplace transform to obtain

$$\mathcal{L}^{-1}\left[\frac{2s-3}{(s-1)(s-2)}\right] = \mathcal{L}^{-1}\left[\frac{1}{s-1}\right] + \mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{t} + e^{2t}, \quad t \geq 0. \blacksquare$$

Problem 45.7

Find $\mathcal{L}^{-1}\left[\frac{4s^2+s+1}{s^3+s}\right]$

Solution.

We factor the denominator and split the rational function into partial fractions:

$$\frac{4s^2+s+1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}.$$

Multiplying both sides by $s(s^2+1)$ and simplifying to obtain

$$\begin{aligned} 4s^2+s+1 =& A(s^2+1) + (Bs+C)s \\ =& (A+B)s^2 + Cs + A. \end{aligned}$$

Equating coefficients of like powers of s we obtain $A+B=4,\ C=1,\ A=1$. Thus, $B=3$. Now finding the inverse Laplace transform to obtain

$$\begin{aligned} \mathcal{L}^{-1}\left(\frac{4s^2+s+1}{s(s^2+1)}\right) =& \mathcal{L}^{-1}\left(\frac{1}{s}\right) + 3\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) \\ =& 1 + 3\cos t + \sin t,\quad t \geq 0 \end{aligned}$$ ■

Problem 45.8

Find $\mathcal{L}^{-1}\left(\frac{s^2+6s+8}{s^4+8s^2+16}\right)$.

Solution.

We factor the denominator and split the rational function into partial fractions:

$$\frac{s^2+6s+8}{(s^2+4)^2} = \frac{B_1s+C_1}{s^2+4} + \frac{B_2s+C_2}{(s^2+4)^2}.$$

Multiplying both sides by $(s^2+4)^2$ and simplifying to obtain

$$s^2+6s+8 = (B_1s+C_1)(s^2+4) + B_2s + C_2$$

$$=B_1s^3 + C_1s^2 + (4B_1 + B_2)s + 4C_1 + C_2.$$

Equating coefficients of like powers of s we obtain $B_1 = 0$, $C_1 = 1$, $B_2 = 6$, and $C_2 = 4$. Now finding the inverse Laplace transform to obtain

$$\mathcal{L}^{-1}\left\{\frac{s^2+6s+8}{(s^2+4)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} + 6\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{1}{(s^2+4)^2}\right\}$$

$$= \frac{1}{2}\sin 2t + 6\,\frac{t}{4}\sin 2t + 4\,\frac{1}{16}[\sin 2t - 2t\cos 2t]$$

$$= \frac{3}{2}t\sin 2t + \frac{3}{4}\sin 2t - \frac{1}{2}t\cos 2t,\quad t \geq 0 \quad \blacksquare$$

Problem 45.9

Use Laplace transform to solve the initial value problem

$$y' + 2y = 26\sin 3t,\ y(0) = 3.$$

Solution.

Taking the Laplace of both sides to obtain

$$\mathbf{L}[y'] + 2\mathbf{L}[y] = 26\mathbf{L}[\sin 3t].$$

Using Table L the last equation reduces to

$$sY(s) - y(0) + 2Y(s) = 26\frac{3}{s^2+9}.$$

Solving this equation for $Y(s)$ we find

$$Y(s) = \frac{3}{s+2} + \frac{78}{(s+2)(s^2+9)}.$$

Using the partial fraction decomposition we can write

$$\frac{1}{(s+2)(s^2+9)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+9}.$$

Multipliying both sides by $(s+2)(s^2+9)$ to obtain

$$\begin{aligned} 1 =& A(s^2+9) + (Bs+C)(s+2) \\ =& (A+B)s^2 + (2B+C)s + 9A + 2C. \end{aligned}$$

Equating coefficients of like powers of s we find $A+B=0,\ 2B+C=0,$ and $9A+2C=1$. Solving this system we find $A = \frac{1}{13}$, $B = -\frac{1}{13}$, and $C = \frac{2}{13}$.

Thus,

$$Y(s) = \frac{9}{\ } - 6\frac{s}{\ } + 4\frac{3}{\ }.$$

$s + 2 \qquad s^2 + 9 \qquad s^2 + 9$

Finally,

$$y(t) = L^{-1}[Y(s)] = 9L^{-1}\left[\frac{1}{s+2}\right] - 6L^{-1}\left[\frac{s}{s^2+9}\right] + 4L^{-1}\left[\frac{3}{s^2+9}\right]$$

$$= 9e^{-2t} - 6\cos 3t + 4\sin 3t,\ t \geq 0$$

∎

Problem 45.10

Use Laplace transform to solve the initial value problem

$$y' + 2y = 4t, \; y(0) = 3.$$

Solution.

Taking the Laplace of both sides to obtain

$$\mathbf{L}[y'] + 2\mathbf{L}[y] = 4\mathbf{L}[t].$$

Using Table L the last equation reduces to

$$sY(s) - y(0) + 2Y(s) = \frac{4}{s^2}.$$

Solving this equation for $Y(s)$ we find

$$Y(s) = \frac{3}{s+2} + \frac{4}{(s+2)s^2}.$$

Using the partial fraction decomposition we can write

$$\frac{1}{(s+2)s^2} = \frac{A}{s+2} + \frac{Bs+C}{s^2}.$$

Multipliying both sides by $(s+2)s^2$ to obtain

$$\begin{aligned} 1 &= As^2 + (Bs+C)(s+2) \\ &= (A+B)s^2 + (2B+C)s + 2C \end{aligned}$$

Equating coefficients of like powers of s we find $A+B=0$, $2B+C=0$, and$2C=1$. Solving this system we find $A=\frac{1}{4}$, $B=-\frac{1}{4}$, and $C=\frac{1}{2}$. Thus,

Finally, 4 1 1

$Y(s) = s +$

$2 - s + 2 s^2 \cdot$

$$y(t) = \mathrm{L}^{-1}[Y(s)] = 4\mathrm{L}^{-1}\left[\frac{1}{s+2}\right] - \mathrm{L}^{-1}\left[\frac{1}{s}\right] + 2\mathrm{L}^{-1}\left[\frac{1}{s^2}\right]$$

$$= 4e^{-2t} - 1 + 2t, \ t \geq 0 \quad \blacksquare$$

Problem 45.11

Use Laplace transform to solve the initial value problem

$$y'' + 3y' + 2y = 6e^{-t},\ y(0) = 1,\ \ y'(0) = 2.$$

Solution.

Taking the Laplace of both sides to obtain

$$\mathbf{L}[y''] + 3\mathbf{L}[y'] + 2\mathbf{L}[y] = 6\mathbf{L}[e^{-t}].$$

Using Table L the last equation reduces to

$$s^2Y(s) - sy(0) - y'(0) + 3(sY(s) - y(0)) + 2Y(s) = \frac{6}{s+1}.$$

Solving this equation for $Y(s)$ we find

$$Y(s) = \frac{s+5}{(s+1)(s+2)} + \frac{6}{(s+2)(s+1)^2} = \frac{s^2+6s+11}{(s+1)^2(s+2)}.$$

Using the partial fraction decomposition we can write

$$\frac{s^2+6s+11}{(s+2)(s+1)^2} = \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{(s+1)^2}.$$

Multipliying both sides by $(s+2)(s+1)^2$ to obtain

$$\begin{aligned} s^2 + 6s + 11 &= A(s+1)^2 + B(s+1)(s+2) + C(s+2) \\ &= (A+B)s^2 + (2A+3B+C)s + A + 2B + 2C \end{aligned}$$

Equating coefficients of like powers of s we find $A + B = 1$, $2A + 3B + C = 6$, and $A + 2B + 2C = 11$. Solving this system we find $A = 3$, $B = \ 2$, and $C = 6$. Thus,

$$, \quad \frac{}{} \quad \frac{3}{} \quad \frac{2\ \ 6}{}$$

Finally ■

$$Y(s) = \qquad \overline{s+2} - \overline{s+1} + \overline{(s+1)^2}\,.$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = 3\mathcal{L}^{-1}\frac{1}{s+2} - 2\mathcal{L}^{-1}\frac{1}{s+1} + 6\mathcal{L}^{-1}\frac{1}{(s+1)^2}$$

$$= 3e^{-2t} - 2e^{-t} + 6te^{-t},\ t \geq 0$$

■

Problem 45.12

Use Laplace transform to solve the initial value problem

$$y'' + 4y = \cos 2t, \quad y(0) = 1, \quad y'(0) = 1.$$

Solution.

Taking the Laplace of both sides to obtain

$$\mathbf{L}[y''] + 4\mathbf{L}[y] = \mathbf{L}[\cos 2t].$$

Using Table L the last equation reduces to

$$s^2 Y(s) - sy(0) - y'(0) + 4Y(s) = \frac{s}{s^2 + 4}.$$

Solving this equation for $Y(s)$ we find

$$Y(s) = \frac{s+1}{s^2+4} + \frac{s}{(s^2+4)^2}.$$

Using Table L again we find

$$y(t) = \mathbf{L}^{-1}\left[\frac{s}{s^2+4}\right] + \frac{1}{2}\mathbf{L}^{-1}\left[\frac{2}{s^2+4}\right] + \mathbf{L}^{-1}\left[\frac{s}{(s^2+4)^2}\right]$$

$$= \cos 2t + \frac{1}{2}\sin 2t + \frac{t}{4}\sin 2t, \quad t \geq 0 \quad \blacksquare$$

Problem 45.13

Use Laplace transform to solve the initial value problem

$$y'' - 2y' + y = e^{2t}, \quad y(0) = 0, \quad y'(0) = 0.$$

Solution.

Taking the Laplace of both sides to obtain

$$\mathbf{L}[y''] - 2\mathbf{L}[y'] + \mathbf{L}[y] = \mathbf{L}[e^{2t}].$$

Using Table L the last equation reduces to

$$s^2Y(s) - sy(0) - y'(0) - 2sY(s) + 2y(0) + Y(s) = \frac{1}{s-2}.$$

Solving this equation for $Y(s)$ we find

$$Y(s) = \frac{1}{(s-1)^2(s-2)}.$$

Using the partial fraction decomposition, we can write

$$Y(s) = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s-2}.$$

Multipliying both sides by $(s-2)(s-1)^2$ to obtain

$$1 = A(s-1)(s-2) + B(s-2) + C(s-1)^2$$

$$= (A+C)s^2 + (-3A + B - 2C)s + 2A - 2B + C$$

Equating coefficients of like powers of s we find $A+C = 0$, $-3A+B-2C = 0$, and $2A - 2B + C = 1$. Solving this system we find $A = -1$, $B = -1$, and $C = 1$. Thus,

$$Y(s) = -\frac{1}{s-1} - \frac{1}{(s-1)^2} + \frac{1}{s-2}.$$

Finally,

$$y(t) = \mathcal{L}^{-1}[Y(s)] = -\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) - \mathcal{L}^{-1}\left(\frac{1}{(s-1)^2}\right) + \mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$$

$$= -e^t - te^t + e^{2t}, \quad t \geq 0 \quad \blacksquare$$

Problem 45.14

Use Laplace transform to solve the initial value problem

$$y'' + 9y = g(t), \; y(0) = 1, \; y'(0) = 3$$

where

Solution.

$$g(t) = \begin{cases} 6, & 0 \le t < \pi \\ 0, & \pi \le t < \infty \end{cases}$$

Taking the Laplace of both sides to obtain

$$\mathbf{L}[y''] + 9\mathbf{L}[y] = \mathbf{L}[g(t)] = 6\mathbf{L}[h(t) - h(t - \pi)].$$

Using Table L the last equation reduces to

$$s^2Y(s) - sy(0) - y'(0) + 9Y(s) = \frac{6}{s} - \frac{6e^{-\pi s}}{s}.$$

Solving this equation for $Y(s)$ we find

$$Y(s) = \frac{s+3}{s^2+9} + \frac{6}{s(s^2+9)}(1 - e^{-\pi s}).$$

Using the partial fraction decomposition, we can write

$$\frac{6}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9}.$$

Multipliying both sides by $s(s^2+9)$ to obtain

$$\begin{aligned} 6 &= A(s^2+9) + (Bs+C)s \\ &= (A+B)s^2 + Cs + 9A \end{aligned}$$

Equating coefficients of like powers of s we find $A + B = 0$, $C = 0$, and$9A = 6$. Solving this system we find $A = \frac{2}{3}$, $B = -\frac{2}{3}$, and $C = 0$. Thus,

$$Y(s) = \frac{s}{s^2+9} + \frac{3}{s^2+9} + (1 - e^{-\pi s})\left(\frac{2}{3}\frac{1}{s} - \frac{2}{3}\frac{s}{s^2+9}\right).$$

Finally,

$$\begin{aligned} y(t) = \mathcal{L}^{-1}[Y(s)] &= \cos 3t + \sin 3t + \frac{2}{3}(1 - \cos 3t) - \frac{2}{3}(1 - \cos 3(t-\pi))h(t-\pi) \\ &= \cos 3t + \sin 3t + \frac{2}{3}(1 - \cos 3t) - \frac{2}{3}(1 + \cos 3t)h(t-\pi),\ t \geq 0 \quad \blacksquare \end{aligned}$$

Problem 45.15

Determine the constants $\alpha, \beta, y_0,$ and y_0' so that $Y(s) = \frac{2s-1}{s^2+s+2}$ is the Laplace transform of the solution to the initial value problem

$$y'' + \alpha y' + \beta y = 0, \;\; y(0) = y_0, \;\; y'(0) = y_0'.$$

Solution.

Taking the Laplace transform of both sides we find

$$s^2 Y(s) - sy_0 - y_0' + \alpha s Y(s) - \alpha y_0 + \beta Y(s) = 0.$$

Solving for $Y(s)$ we find

$$Y(s) = \frac{sy_0 + (y_0' + \alpha y_0)}{s^2 + \alpha s + \beta} = \frac{2s - 1}{s^2 + s + 2}.$$

By identification we find $\alpha = 1,\ \beta = 2,\ y_0 = 2,$ and $y_0' = -3$ ■

Problem 45.16

Determine the constants $\alpha, \beta, y_0,$ and y_0' so that $Y(s) = \frac{s}{(s+1)^2}$ is the Laplace transform of the solution to the initial value problem

$$y'' + \alpha y' + \beta y = 0,\ y(0) = y_0,\ y'(0) = y_0'.$$

Solution.

Taking the Laplace transform of both sides we find

$$s^2 Y(s) - sy_0 - y_0' + \alpha s Y(s) - \alpha y_0 + \beta Y(s) = 0.$$

Solving for $Y(s)$ we find

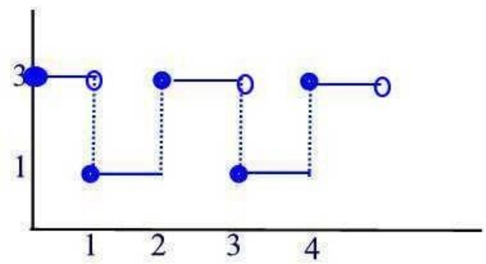
$$Y(s) = \frac{sy_0 + (y_0' + \alpha y_0)}{s^2 + \alpha s + \beta} = \frac{s}{s^2 + 2s + 1}.$$

By identification we find $\alpha = 2,\ \beta = 1,\ y_0 = 1,$ and $y_0' = -2$ ■

Section 46

Problem 46.1

Find the Laplace transform of the periodic function whose graph is shown.



Solution.

The function is of period $T = 2$. Thus,

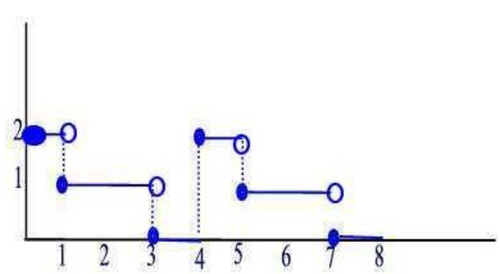
$$3\int_0^1 e^{-st}dt + \int_1^2 e^{-st}dt = -\frac{3}{s}e^{-st}\Big|_0^1 - \frac{e^{-st}}{s}\Big|_1^2 = \frac{1}{s}(3-2e^{-s}-e^{-2s}).$$

Hence,

$$L[f(t)] = \frac{3-2e^{-s}-e^{-2s}}{s(1-e^{-2s})} \blacksquare$$

Problem 46.2

Find the Laplace transform of the periodic function whose graph is shown.



Solution.

The function is of period $T = 4$. Thus,

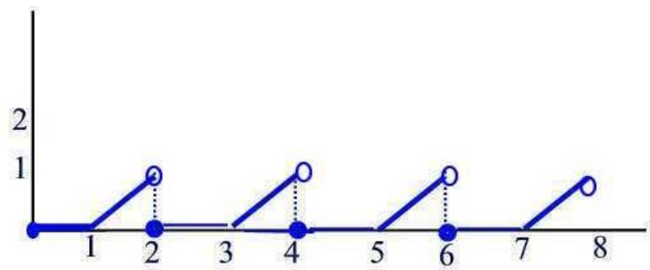
$$2\int_0^1 e^{-st}dt + \int_1^3 e^{-st}dt = -\frac{2}{s}e^{-st}\Big|_0^1 - \frac{e^{-st}}{s}\Big|_1^3 = \frac{1}{s}(2 - e^{-s} - e^{-3s}).$$

Hence,

$$\mathcal{L}[f(t)] = \frac{2 - e^{-s} - e^{-3s}}{s(1 - e^{-4s})} \blacksquare$$

Problem 46.3

Find the Laplace transform of the periodic function whose graph is shown.



Solution.

The function is of period $T = 2$. Thus,

$$\int_1^2 (t-1)e^{-st}dt = \int_1^2 te^{-st}dt - \int_1^2 e^{-st}dt$$

$$= \left[-\frac{t}{s}e^{-st} - \frac{e^{-st}}{s^2} + \frac{e^{-st}}{s}\right]_1^2$$

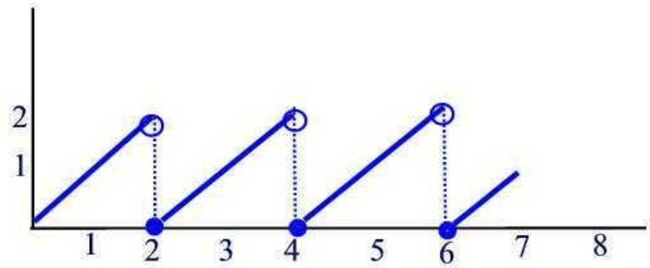
$$= -\frac{e^{-s}}{s^2}[(1+s)e^{-s} - 1]$$

Hence,

$$\mathcal{L}[f(t)] = \frac{e^{-s}}{s^2(1 - e^{-2s})}[1 - (s+1)e^{-s}] \blacksquare$$

Problem 46.4

Find the Laplace transform of the periodic function whose graph is shown.


2
1
1
2
3
4
5
6
7
8

Solution.
The function is of period $T = 2$. Thus,

$$\int_0^2 te^{-st}dt = \left[-\frac{1}{s^2}(st+1)e^{-st}\right]_0^2 = \frac{1}{s^2}[(2s+1)e^{-2s} - 1].$$

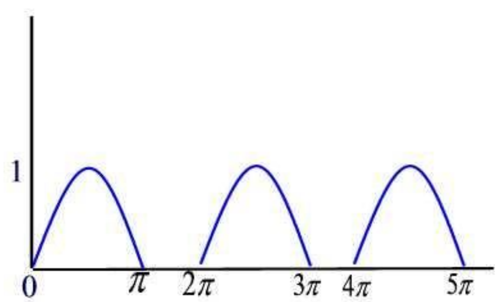
Hence,

$$L[f(t)] = \frac{1}{s^2(1-e^{-2s})}[1 - (2s+1)e^{-2s}] \quad \blacksquare$$

Problem 46.5
State the period of the function $f(t)$ and find its Laplace transform where

$$f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ 0, & \pi \le t < 2\pi \end{cases} \qquad f(t+2\pi) = f(t), \ t \ge 0.$$

Solution.
The graph of $f(t)$ is shown below.



The function $f(t)$ is of period $T = 2\pi$. The Laplace transform of $f(t)$ is

$$L[f(t)] = \frac{\int_0^\pi \sin te^{-st}dt}{1 - e^{-2\pi s}}$$

Using integration by parts twice we find

$$\int \sin t e^{-st} \, dt = -\frac{e^{-st}}{1+s^2} (\cos t + s \sin t)$$

Thus,

$$\int_0^\pi \sin t e^{-st}dt = \left[-\frac{e^{-st}}{1+s^2}(\cos t + s\sin t)\right]_0^\pi$$

$$= \frac{e^{-\pi s}}{1+s^2} + \frac{1}{1+s^2}$$

$$= \frac{1+e^{-\pi s}}{1+s^2}$$

Hence,

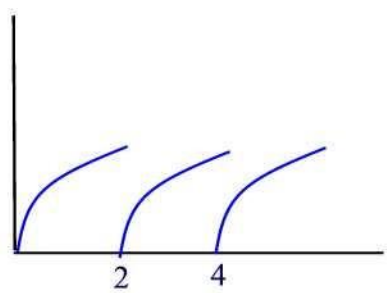
$$\mathcal{L}[f(t)] = \frac{1+e^{-\pi s}}{(1+s^2)(1-e^{-2\pi s})} \blacksquare$$

Problem 46.6

State the period of the function $f(t) = 1 - e^{-t}$, $0 \le t < 2$, $f(t+2) = f(t)$, and find its Laplace transform.

Solution.

The graph of $f(t)$ is shown below



The function is periodic of period $T = 2$. Its Laplace transform is

$$\mathcal{L}[f(t)] = \frac{\int_0^2 (1-e^{-t})e^{-st}dt}{1-e^{-2s}}.$$

But

2

$$\int_0^2 (1-e^{-t})e^{-st}dt = \left[\frac{e^{-st}}{-s}\right]_0^2 - \left[\frac{e^{-(s+1)t}}{s+1}\right]_0^2 = \frac{1}{s}(1-e^{-2s}) - \frac{1}{s+1}(1-e^{-2(s+1)}).$$

Hence,

$$\mathrm{L}[f(t)] = \frac{1}{s} - \frac{1-e^{-2(s+1)}}{(s+1)(1-e^{-2s})} \blacksquare$$

Problem 46.7
Using Example 46.3 find

$$\mathcal{L}^{-1}\left(\frac{s^2-s}{s^3}+\frac{e^{-s}}{s(1-e^{-s})}\right).$$

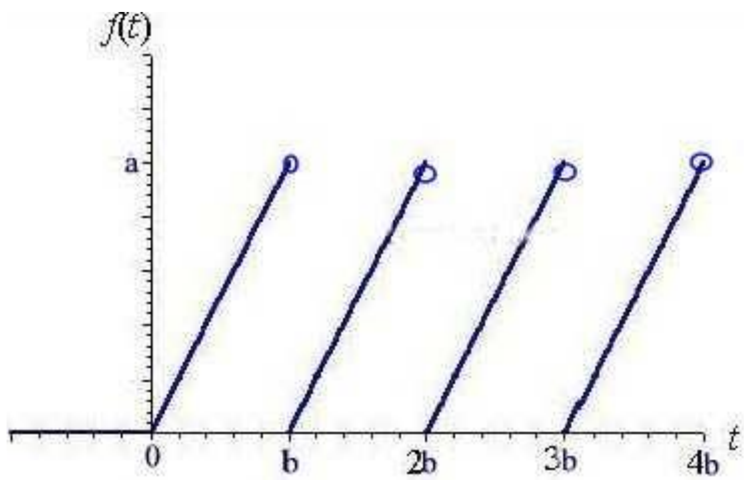
Solution.
Note first that

$$\frac{s^2-s}{s^3}+\frac{e^{-s}}{s(1-e^{-s})}=\frac{1}{s}-\left(\frac{1}{s^2}-\frac{se^{-s}}{s^2(1-e^{-s})}\right).$$

Using Example 23.3, we find

$$f(t)=1-g(t)$$

where $g(t)$ is the sawtooth function shown below



Problem 46.8
An object having mass m is initially at rest on a frictionless horizontal surface. At time $t=0$, a periodic force is applied horizontally to the object, causing it to move in the positive x-direction. The force, in newtons, is given by

$$f(t) = \begin{cases} f_0, & 0 \le t \le \frac{T}{2} \\ 0, & \frac{T}{2} < t < T \end{cases} \qquad f(t+T) = f(t), \quad t \ge 0.$$

The initial value problem for the horizontal position, $x(t)$, of the object is

$$mx''(t) = f(t), \quad x(0) = x'(0) = 0.$$

(a) Use Laplace transforms to determine the velocity, $v(t) = x'(t)$, and the position, $x(t)$, of the object.
(b) Let $m = 1$ *kg*, $f_0 = 1$ *N*, and $T = 1$ *sec*. What is the velocity, v, and position, x, of the object at $t = 1.25$ sec?

Solution.

(a) Taking Laplace transform of both sides we find $ms^2X(s) = \frac{f_0 \int_0^{\frac{T}{2}} e^{-st}dt}{1-e^{-sT}} = \frac{f_0}{s}\frac{1-e^{-s\frac{T}{2}}}{1-e^{-sT}}$. Solving for

$$X(s) = \frac{f_0}{m} \cdot \frac{1}{s^3} \cdot \frac{1}{1+e^{-s\frac{T}{2}}}.$$

Also,

$$V(s) = \mathcal{L}[v(t)] = sX(s) = \frac{f_0}{m} \cdot \frac{1}{s^2} \cdot \frac{1}{1+e^{-s\frac{T}{2}}} = \frac{1}{m} \cdot \frac{1}{s} \cdot \frac{1}{s(1+e^{-s\frac{T}{2}})}.$$

Hence, by Example 46.1 and Table L we can write

$$v(t) = \frac{1}{m}\int_0^t f(u)du.$$

Since $X(s) = \frac{1}{m}\frac{1}{s^2}\frac{f_0}{s(1+e^{-s\frac{T}{2}})} = \frac{1}{m}\mathcal{L}[t]\mathcal{L}[f(t)] = \frac{1}{m}\mathcal{L}[t * f(t)]$ we have

$$x(t) = \frac{1}{m}(t * f(t)) = \frac{1}{m}\int_0^t (t-u)f(u)du.$$

(b) We have $x(1.25) = \int^{1}(\frac{5}{4}-u)du + \int^{\frac{5}{4}}(\frac{5}{4}-w)dw = \frac{17}{[illegible]}$ meters and $v(1.25) =$

0

$$\int 5_{\bar{4}} f(u)du = \int 1_{\bar{2}}\, dt + \int 5_{\bar{4}}^{\bar{2}\ 4} dt^{\bar{4}\ 4} = \frac{3}{4} \text{ m/sec} \quad 32 \quad \blacksquare$$

Problem 46.9

Consider the initial value problem

$$ay'' + by' + cy = f(t), \quad y(0) = y'(0) = 0, \quad t > 0.$$

Suppose that the transfer function of this system is given by $\Phi(s) = \frac{1}{2s^2+5s+2}$.

(a) What are the constants $a, b,$ and c?

(b) If $f(t) = e^{-t}$, determine $F(s)$, $Y(s)$, and $y(t)$.

Solution.

(a) Taking the Laplace transform of both sides we find $as^2Y(s) + bsY(s) + cY(s) = F(s)$ or

$$\Phi(s) = \frac{Y(s)}{F(s)} = \frac{1}{as^2 + bs + c} = \frac{1}{2s^2 + 5s + 2}.$$

By identification we find $a = 2$, $b = 5$, and $c = 2$.

(b) If $f(t) = e^{-t}$ then $F(s) = \mathcal{L}[e^{-t}] = \frac{1}{s+1}$. Thus,

$$Y(s) = \Phi(s)F(s) = \frac{1}{(s+1)(2s^2+5s+2)}.$$

Using partial fraction decomposition

$$\frac{1}{(s+1)(2s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{2s+1} + \frac{C}{s+2}$$

Multiplying both sides by $s+1$ and setting $s = -1$ we find $A = -1$. Next, multiplying both sides by $2s+1$ and setting $s = -\frac{1}{2}$ we find $B = \frac{4}{3}$. Similarly, multiplying both sides by $s+2$ and setting $s = -2$ we find $C = \frac{1}{3}$. Thus,

$$\begin{aligned} y(t) &= -\mathcal{L}^{-1}\left[\frac{1}{s+1}\right] + \frac{2}{3}\mathcal{L}^{-1}\left[\frac{1}{s+\frac{1}{2}}\right] + \frac{1}{3}\mathcal{L}^{-1}\left[\frac{1}{s+2}\right] \\ &= -e^{-t} + \frac{2}{3}e^{-\frac{t}{2}} + \frac{1}{3}e^{-2t}, \quad t \geq 0 \quad \blacksquare \end{aligned}$$

Problem 46.10

Consider the initial value problem

$$ay'' + by' + cy = f(t), \quad y(0) = y'(0) = 0, \quad t > 0$$

Suppose that an input $f(t) = t$, when applied to the above system produces the output $y(t) = 2(e^{-t} \quad 1) + t(e^{-t} + 1), \quad t \quad 0.$
(a) What is the system transfer function? $\geq$
(b) What will be the output if the Heaviside unit step function $f(t) = h(t)$ is applied to the system?

Solution.

(a) Since $f(t) = t$ we find $F(s) = \frac{1}{s^2}$. Aslo, $Y(s) = L[y(t)] = L[2e^{-t} - 2 +$

$te^{-t} + t] = \frac{2}{s+1} - \frac{2}{s} + \frac{1}{(s+1)^2} + \frac{1}{s^2} = \frac{1}{s^2(s+1)^2}$. But $\Phi(s) = \frac{Y(s)}{F(s)} = \frac{1}{(s+1)^2}$.

(b) If $f(t) = h(t)$ then $F(s) = \frac{1}{s}$ and $Y(s) = \Phi(s)F(s) = \frac{1}{s(s+1)^2}$. Using partial fraction decomposition we find

$$\frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$1 = A(s+1)^2 + Bs(s+1) + Cs$$

$$1 = (A+B)s^2 + (2A+B+C)s + A$$

Equating coefficients of like powers of s we find $A = 1,\ B = -1,$ and $C = -1$. Therefore,

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

and

$$y(t) = \mathcal{L}^{-1}[Y(s)] = 1 - e^{-t} - te^{-t},\ t \geq 0 \quad \blacksquare$$

Problem 46.11

Consider the initial value problem

$$y'' + y' + y = f(t), \quad y(0) = y'(0) = 0,$$

where

$$f(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ -1, & 1 < t < 2 \end{cases} \qquad f(t+2) = f(t)$$

(a) Determine the system transfer function $\Phi(s)$.

(b) Determine $Y(s)$.

Solution.

(a) Taking the Laplace transform of both sides we find

$$s^2Y(s) + sY(s) + Y(s) = F(s)$$

so that

$$\Phi(s) = \frac{Y(s)}{F(s)} = \frac{1}{s^2 + s + 1}.$$

(b) But

$$\int_0^2 f(t)e^{-st}dt = \int_0^1 e^{-st}dt - \int_1^2 e^{-st}dt$$

$$= \left.\frac{e^{-st}}{-s}\right|_0^1 - \left.\frac{e^{-st}}{-s}\right|_1^2$$

$$= \frac{1}{s}(1-e^{-s}) + \frac{1}{s}(e^{-2s}-e^{-s})$$

$$= \frac{(1-e^{-s})^2}{s}$$

Hence,

$$F(s) = \frac{(1-e^{-s})^2}{s(1-e^{-2s})} = \frac{(1-e^{-s})}{s(1+e^{-s})}$$

and

$$Y(s) = \Phi(s)F(s) = \frac{(1-e^{-s})}{s(1+e^{-s})(s^2+s+1)} \blacksquare$$

Problem 46.12

Consider the initial value problem

$$y''' - 4y = e^t + t, \quad y(0) = y'(0) = y''(0) = 0.$$

(a) Determine the system transfer function $\Phi(s)$.
(b) Determine $Y(s)$.

Solution.

(a) Taking Laplace transform of both sides we find

$$s^3Y(s) - 4Y(s) = F(s).$$

Thus,

$$\Phi(s) = \frac{Y(s)}{F(s)} = \frac{1}{s^3 - 4}.$$

(b) We have

$$F(s) = \mathrm{L}[e^t + t] = \frac{1}{s-1} + \frac{1}{s^2} = \frac{s^2 + s - 1}{(s-1)s^2}.$$

Hence,

$$Y(s) = \frac{s^2 + s - 1}{s^2(s-1)(s^3 - 4)} \quad \blacksquare$$

Problem 46.13

Consider the initial value problem

$$y'' + by' + cy = h(t), \quad y(0) = y_0, \; y'(0) = y_0', \; t > 0.$$

Suppose that $L[y(t)] = Y(s) = \frac{s^2+2s+1}{s^3+3s^2+2s}$. Determine the constants $b, \; c, \; y_0$, and y_0'.

Solution.

Take the Laplace transform of both sides to obtain

$$s^2Y(s) - sy_0 - y_0' + bsY(s) - by_0 + cY(s) = \frac{1}{s}.$$

Solve to find

$$Y(s) = \frac{s^2y_0 + s(y_0' + by_0) + 1}{s^3 + bs^2 + cs}$$

$$= \frac{s^2 + 2s + 1}{s^3 + 3s^2 + 2s}.$$

By comparison we find $b = 3, \; c = 2, \; y_0 = 1$, and $y_0' + by_0 = 2$ or $y_0' = -1$ ■

Section 47

Problem 47.1

Consider the functions $f(t) = g(t) = h(t), \ t \geq 0$ where $h(t)$ is the Heaviside unit step function. Compute $f * g$ in two different ways.

(a) By directly evaluating the integral.

(b) By computing $\mathcal{L}^{-1}[F(s)G(s)]$ where $F(s) = \mathcal{L}[f(t)]$ and $G(s) = \mathcal{L}[g(t)]$.

Solution.

(a) We have

$$(f * g)(t) = \int_0^t f(t-s)g(s)ds = \int_0^t h(t-s)h(s)ds = \int_0^t ds = t, \ t \geq 0.$$

(b) Since $F(s) = G(s) = \mathcal{L}[h(t)] = \frac{1}{s}$ we have $(f * g)(t) = \mathcal{L}^{-1}[F(s)G(s)] = \mathcal{L}^{-1}[\frac{1}{s^2}] = t, \ t \geq 0$ ■

Problem 47.2

Consider the functions $f(t) = e^t$ and $g(t) = e^{-2t}, \ t \geq 0$. Compute $f * g$ in two different ways.

(a) By directly evaluating the integral.

(b) By computing $\mathcal{L}^{-1}[F(s)G(s)]$ where $F(s) = \mathcal{L}[f(t)]$ and $G(s) = \mathcal{L}[g(t)]$.

Solution.

(a) We have

$$(f * g)(t) = \int_0^t f(t-s)g(s)ds = \int_0^t e^{(t-s)}e^{-2s}ds$$

$$= e^t \int_0^t e^{-3s}ds = \left[\frac{e^{(t-3s)}}{-3}\right]_0^t$$

$$e^t - e^{-2t}$$

$= \frac{\qquad}{3}, \; t \geq 0.$

(b) Since $F(s) = L[e^t] = \frac{1}{s-1}$ and $G(s) = L[e^{-2t}] = \frac{1}{s+2}$ we find $(f * g)(t) = L^{-1}[F(s)G(s)] = L^{-1}[\frac{1}{(s-1)(s-2)}]$. Using partial fractions decomposition we find

$$\frac{1}{(s-1)(s+2)} = \frac{1}{3}\left(\frac{1}{s-1} - \frac{1}{s+2}\right).$$

Thus,

$$(f*g)(t) = \mathcal{L}^{-1}[F(s)G(s)] = \frac{1}{3}\mathcal{L}^{-1}\left[\frac{1}{s-1}\right] - \frac{1}{3}\mathcal{L}^{-1}\left[\frac{1}{s+2}\right] = \frac{e^t - e^{-2t}}{3}, \; t \geq 0 \;\blacksquare$$

Problem 47.3

Consider the functions $f(t) = \sin t$ and $g(t) = \cos t$, $t \geq 0$. Compute $f * g$ in two different ways.

(a) By directly evaluating the integral.

(b) By computing $\mathcal{L}^{-1}[F(s)G(s)]$ where $F(s) = \mathcal{L}[f(t)]$ and $G(s) = \mathcal{L}[g(t)]$.

Solution.

(a) Using the trigonometric identity $2 \sin p \cos q = \sin(p + q) + \sin(p - q)$ we find that $2 \sin(t - s) \cos s = \sin t + \sin(t - 2s)$. Hence,

$$\begin{aligned}
(f * g)(t) &= \int_0^t f(t-s)g(s)ds = \int_0^t \sin(t-s)\cos s\,ds \\
&= \frac{1}{2}\left[\int_0^t \sin t\,ds + \int_0^t \sin(t-2s)ds\right] \\
&= \frac{t\sin t}{2} + \frac{1}{4}\int_{-t}^{t} \sin u\,du \\
&= \frac{t\sin t}{2}, \; t \geq 0.
\end{aligned}$$

(b) Since $F(s) = \mathcal{L}[\sin t] = \frac{1}{s^2+1}$ and $G(s) = \mathcal{L}[\cos t] = \frac{s}{s^2+1}$ we find

$$(f * g)(t) = \mathcal{L}^{-1}[F(s)G(s)] = \mathcal{L}^{-1}\left[\frac{s}{(s^2+1)^2}\right] = \frac{t}{2}\sin t, \; t \geq 0 \;\blacksquare$$

Problem 47.4

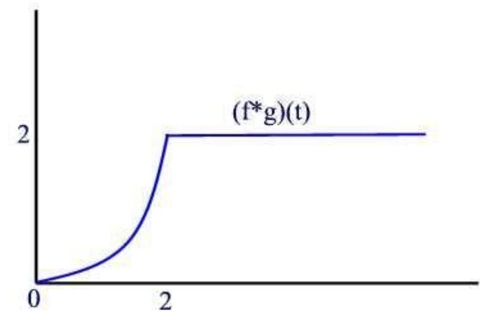
Compute and graph $f * g$ where $f(t) = h(t)$ and $g(t) = t[h(t) - h(t - 2)]$.

Solution.

Since $f(t) = h(t)$, $F(s) = \frac{1}{s}$. Similarly, since $g(t) = th(t) - th(t-2) = th(t) - (t-2)h(t-2) - 2h(t-2)$, $G(s) = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s}$. Thus, $F(s)G(s) = \frac{1}{s^3} - \frac{e^{-2s}}{s^3} - \frac{2e^{-2s}}{s^2}$. It follows that

$$(f * g)(t) = \frac{t^2}{2} - \frac{(t-2)^2}{2} h(t-2) - 2(t-2)h(t-2),\ t \geq 0.$$

The graph of $(f * g)(t)$ is given below ∎



Problem 47.5

Compute and graph $f * g$ where $f(t) = h(t) - h(t-1)$ and $g(t) = h(t-1) - 2h(t-2)$.

Solution.

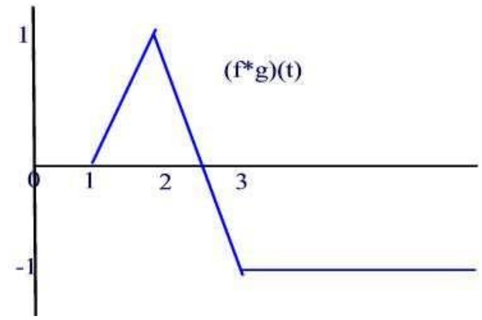
Since $f(t) = h(t) - h(t-1)$, $F(s) = \frac{1}{s} - \frac{e^{-s}}{s}$. Similarly, since $g(t) = h(t-1) - 2h(t-2)$, $G(s) = \frac{e^{-s}}{s} - \frac{2e^{-2s}}{s}$. Thus,

$$F(s)G(s) = \frac{e^{-s} - 3e^{-2s} + 2e^{-3s}}{s^2} = \frac{e^{-s}}{s^2} - 3\frac{e^{-2s}}{s^2} + 2\frac{e^{-3s}}{s^2}$$

It follows that

$$(f * g)(t) = (t-1)h(t-1) - 3(t-2)h(t-2) + 2(t-3)h(t-3),\ t \geq 0.$$

The graph of $(f * g)(t)$ is given below ■


1
(f*g)(t)
0
1
2
3
-1

Problem 47.6

Compute $t * t * t$.

Solution.

By the convolution theorem we have $\mathcal{L}[t * t * t] = (\mathcal{L}[t])^3 = \left(\frac{1}{s^2}\right)^3 = \frac{1}{s^6}$. Hence,

$$t * t * t = \mathcal{L}^{-1}\left[\frac{1}{s^6}\right] = \frac{t^5}{5!} = \frac{t^5}{120}, \; t \geq 0 \quad \blacksquare$$

Problem 47.7

Compute $h(t) * e^{-t} * e^{-2t}$.

Solution.

By the convolution theorem we have $\mathcal{L}[h(t)*e^{-t}*e^{-2t}] = \mathcal{L}[h(t)]\mathcal{L}[e^{-t}]\mathcal{L}[e^{-2t}] = \frac{1}{s} \cdot \frac{1}{s+1} \cdot \frac{1}{s+2}$. Using the partial fractions decomposition we can write

$$\frac{1}{s(s+1)(s+2)} = \frac{1}{2s} - \frac{1}{s+1} + \frac{1}{2} \cdot \frac{1}{s+2}.$$

Hence,

$$h(t) * e^{-t} * e^{-2t} = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}, \; t \geq 0 \quad \blacksquare$$

Problem 47.8

Compute $t * e^{-t} * e^{t}$.

Solution.

By the convolution theorem we have $\mathcal{L}[t * e^{-t} * e^{t}] = \mathcal{L}[t]\mathcal{L}[e^{-t}]\mathcal{L}[e^{t}] = \frac{1}{s^2} \cdot \frac{1}{s+1} \cdot \frac{1}{s-1}$. Using the partial fractions decomposition we can write

$$\frac{1}{s^2(s+1)(s-1)} = -\frac{1}{s^2} + \frac{1}{2} \cdot \frac{1}{s-1} - \frac{1}{2} \cdot \frac{1}{s+1}.$$

Hence,

$$t * e^{-t} * e^{t} = -t + \frac{e^t}{2} - \frac{e^{-t}}{2}$$

Problem 47.9

e^{-t} $\quad {}_2$, $t \geq 0$ — ■

$\overline{\quad n \text{ functions} \quad}$

Suppose it is known that $h(t) \quad h(t) * \cdots * h(t) = Ct^8$. Determine the constants C and the positive integer n.

Solution.

We know that $L[\overbrace{h(t) * h(t) * \cdots * h(t)}^{n \text{ functions}}] = (L[h(t)])^n = \frac{1}{s^n}$ so that $L^{-1}[\frac{1}{s^n}] = \frac{t^{n-1}}{(n-1)!} = Ct^8$. It follows that $n = 9$ and $C = \frac{1}{8!}$ ■

Problem 47.10

Use Laplace transform to solve for $y(t)$:

$$\int_0^t \sin(t-\lambda) y(\lambda) d\lambda = t^2.$$

Solution.

Note that the given equation reduces to $\sin t * y(t) = t^2$. Taking Laplace transform of both sides we find $\frac{Y(s)}{s^2+1} = \frac{2}{s^3}$. This implies $Y(s) = \frac{2(s^2+1)}{s^3} = \frac{2}{s} + \frac{2}{s^3}$. Hence, $y(t) = L^{-1}[\frac{2}{s} + \frac{2}{s^3}] = 2 + t^2$, $t \geq 0$ ■

Problem 47.11

Use Laplace transform to solve for $y(t)$:

$$y(t) - \int_0^t e^{(t-\lambda)} y(\lambda) d\lambda = t.$$

Solution.

Note that the given equation reduces to $e^t * y(t) = y(t) - t$. Taking Laplace transform of both sides we find $\frac{Y(s)}{s-1} = Y(s) - \frac{1}{s^2}$. Solving for $Y(s)$ we find $Y(s) = \frac{s-1}{s^2(s-2)}$. Using partial fractions decomposition we can write

$\frac{1}{}$ $\frac{1}{}$ $\frac{1}{}$

$$\frac{s-1}{s^2(s-2)} = \frac{-\frac{1}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{1}{4}}{(s-2)}.$$

Hence,

$$y(t) = -\frac{1}{4} + \frac{t}{2} + \frac{1}{4}e^{2t}, \ t \geq 0 \quad \blacksquare$$

Problem 47.12

Use Laplace transform to solve for $y(t)$:

$$t * y(t) = t^2(1 - e^{-t}).$$

Solution.

Taking Laplace transform of both sides we find $\frac{Y(s)}{s^2} = \frac{2}{s^3} - \frac{2}{(s+1)^3}$. This implies $Y(s) = \frac{2}{s} - \frac{2s^2}{(s+1)^3}$. Using partial fractions decomposition we can write

$$\frac{s^2}{(s+1)^3} = \frac{1}{s+1} - \frac{2}{(s+1)^2} + \frac{1}{(s+1)^3}.$$

Hence,

$$y(t) = 2 - 2\left(e^{-t} - 2te^{-t} + \frac{t^2}{2}e^{-t}\right) = 1 - \left(1 - 2t + \frac{t^2}{2}\right)e^{-t}, \ t \ge 0 \qquad \blacksquare$$

Problem 47.13

Solve the following initial value problem.

$$y' - y = \int_0^t (t-\lambda)e^{\lambda}d\lambda, \quad y(0) = -1.$$

Solution.

Note that $y' - y = t * e^t$. Taking Lalplace transform of both sides we find $sY - (-1) - Y = \frac{1}{s^2} \cdot \frac{1}{s-1}$. This implies that $Y(s) = -\frac{1}{s-1} + \frac{1}{s^2(s-1)^2}$. Using partial fractions decomposition we can write

$$\frac{1}{s^2(s-1)^2} = \frac{2}{s} + \frac{1}{s^2} - \frac{2}{s-1} + \frac{1}{(s-1)^2}.$$

Thus,

$$Y(s) = -\frac{1}{s-1} + \frac{2}{s} + \frac{1}{s^2} - \frac{2}{s-1} + \frac{1}{(s-1)^2} = \frac{2}{s} + \frac{1}{s^2} - \frac{3}{s-1} + \frac{1}{(s-1)^2}.$$

Finally,

$$y(t) = 2 + t - 3e^t + te^t, \ t \ge 0 \quad \blacksquare$$

Section 48

Problem 48.1

Evaluate

(a) $\int_0^3 (1+e^{-t})\delta(t-2)dt.$

(b) $\int_{-2}^1 (1+e^{-t})\delta(t-2)dt.$

Solution.

(a) $\int_0^3 (1+e^{-t})\delta(t-2)dt = 1+e^{-2}.$

(b) $\int_{-2}^1 (1+e^{-t})\delta(t-2)dt = 0$ since 2 lies outside the integration interval ■

Problem 48.2

Let $f(t)$ be a function defined and continuous on $0 \le t < \infty$. Determine

$$(f*\delta)(t) = \int_0^t f(t-s)\delta(s)ds.$$

Solution.

Let $g(s) = f(t-s)$. Then

$$(f*\delta)(t) = \int_0^t f(t-s)\delta(s)ds = \int_0^t g(s)\delta(s)ds$$

$$= g(0) = f(t)$$ ■

Problem 48.3

Determine a value of the constant t_0 such that $\int_0^1 \sin^2[\pi(t-t_0)]\delta(t-\frac{1}{2})dt = \frac{3}{4}$.

Solution.

We have $\int_0^1 \sin^2[\pi(t-t_0)]\delta(t-$

0 0 2 4

$$) dt = \frac{1}{2} \quad \frac{3}{4}$$

$$\sin^2 \pi \left(\frac{1}{2} - t_0\right) = \frac{3}{4}$$

$$\sin \pi \left(\frac{1}{2} - t_0\right) = \pm \frac{\sqrt{3}}{2}.$$

Thus, a possible value is when $\pi \left(\frac{1}{2} - t_0\right) = \frac{\pi}{3}$ Solving for t_0 we find $t_0 = \frac{1}{6}$ ■

Problem 48.4

If $\int_1^5 t^n\delta(t-2)dt = 8$, what is the exponent n?

Solution.

We have $\int_1^5 t^n\delta(t-2)dt = 2^n = 8$. Thus, $n = 3$ ■

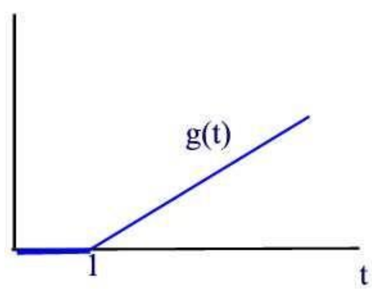
Problem 48.5

Sketch the graph of the function $g(t)$ which is defined by $g(t) = \int_0^t\int_0^s \delta(u-1)duds,\ 0 \le t < \infty$.

Solution.

Note first that $\int_0^s \delta(u-1)du = 1$ if $s > 1$ and 0 otherwise. Hence,

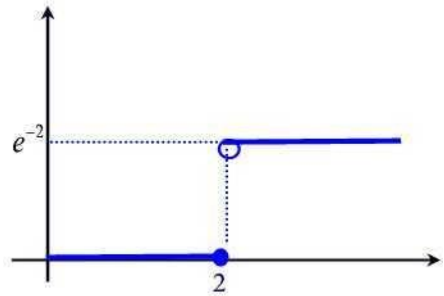
$$g(t) = \begin{cases} 0, & \text{if } t \le 1 \\ \int_1^t h(s-1)ds = t-1, & \text{if } t > 1 \end{cases}$$ ■



Problem 48.6

0 0

Determine the constants α



The graph of the function $g(t) = {}^{t}\, e^{\alpha t}\delta(t - t\ )dt,\ 0 \leq t < \infty$ is shown.

Solution.

Note that
$$g(t) = \begin{cases} 0, & 0 \le t \le t_0 \\ e^{\alpha t_0}, & t_0 < t < \infty \end{cases}$$

It follows that $t_0 = 2$ and $\alpha = -1$ ■

Problem 48.7

(a) Use the method of integarting factor to solve the initial value problem $y' \quad y = h(t),\ y(0) = 0.$

(b) Use the Laplace transform to solve the initial value problem $\varphi' \quad \varphi = \delta(t),\ \varphi(0) = 0.$

(c) Evaluate the convolution $\varphi \quad h(t)$ and compare the resulting function with the solution obtained in part(a).

Solution.

(a) Using the method of integrating factor we find, for $t \geq 0$,

$$y' - y = h(t)$$
$$(e^{-t}y)' = e^{-t}$$
$$e^{-t}y = -e^{-t} + C$$
$$y = -1 + Ce^{t}$$
$$y = -1 + e^{t}.$$

(b) Taking Laplace of both sides we find $s\Phi - \Phi = 1$ or $\Phi(s) = \frac{1}{s-1}$. Thus, $\varphi(t) = e^t$.

(c) We have

$$(\varphi * h)(t) = \int_0^t e^{(t-s)}h(s)ds = \int_0^t e^{(t-s)}ds = \quad 1 + e^t$$ ■

Problem 48.8

Solve the initial value problem

$$y' + y = 2 + \delta(t-1),\ \ y(0) = 0,\ \ 0 \le t \le 6.$$

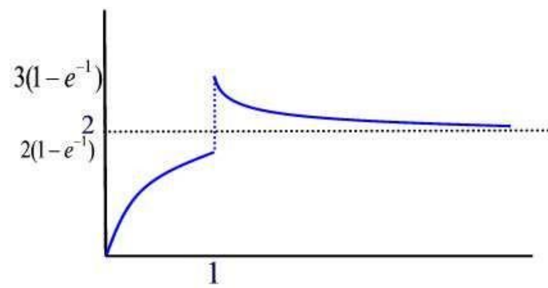
Graph the solution on the indicated interval.

Solution.

Taking Laplace of both sides to obtain $sY + Y = \frac{2}{s} + e^{-s}$. Thus, $Y(s) = \frac{2}{s(s+1)} + \frac{e^{-s}}{s+1} = \frac{2}{s} - \frac{2}{s+1} + \frac{e^{-s}}{s+1}$. Hence,

$$y(t) = \begin{cases} 2 - 2e^{-t}, & t < 1 \\ 2 + (e-2)e^{-t}, & t \geq 1 \end{cases}$$

■



Problem 48.9

Solve the initial value problem

$$y'' = \delta(t-1) - \delta(t-3), \quad y(0) = 0, \quad y'(0) = 0, \quad 0 \leq t \leq 6.$$

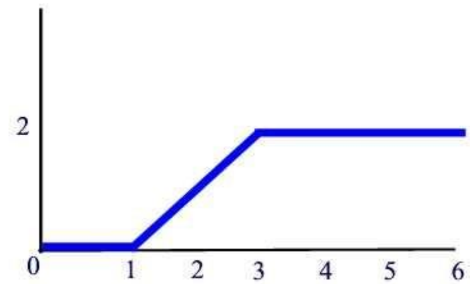
Graph the solution on the indicated interval.

Solution.

Taking Laplace of both sides to obtain $s^2 Y = e^{-s} - e^{-3s}$. Thus, $Y(s) = \frac{e^{-s}}{s^2} - \frac{e^{-3s}}{s^2}$. Hence,

$$y(t) = (t-1)h(t-1) - (t-3)h(t-3)$$

■


2
0
1
2
3
4
5
6

Problem 48.10

Solve the initial value problem

$$y'' - 2y' = \delta(t-1), \quad y(0) = 1, \quad y'(0) = 0, \quad 0 \le t \le 2.$$

Graph the solution on the indicated interval.

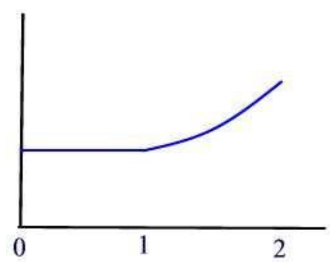
Solution.

Taking Laplace transform of both sides and using the initial conditions we find

$$s^2Y - s - 2(sY - 1) = e^{-s}.$$

Solving for s we find $Y(s) = \frac{1}{s} + \frac{e^{-s}}{s(s-2)} = \frac{1}{s} - \frac{e^{-s}}{2s} + \frac{e^{-s}}{s-2}$. Hence,

$$y(t) = 1 - \frac{1}{2}h(t-1) + \frac{1}{2}e^{2(t-1)}h(t-1) \quad \blacksquare$$



Problem 48.11

Solve the initial value problem

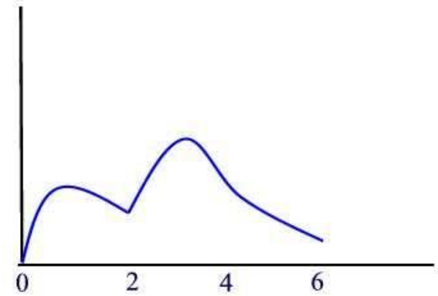
$$y'' + 2y' + y = \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1, \quad 0 \le t \le 6.$$

Graph the solution on the indicated interval.

Solution.

Taking Laplace transform of both sides to obtain $s^2Y - 1 + 2sY + Y = e^{-2s}$.

Solving for $Y(s)$ we find $Y(s) = \frac{1}{(s+1)^2} + \frac{e^{-2s}}{(s+1)^2}$. Therefore, $y(t) = te^{-t} + (t-2)e^{-(t-2)}h(t-2)$ $\blacksquare$


0
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Section 49

Problem 49.1
Find $\mathcal{L}[\mathbf{y}(t)]$ where

$$\mathbf{y}(t) = \frac{d}{dt}\begin{bmatrix} e^{-t}\cos 2t \\ 0 \\ t + e^{t} \end{bmatrix}$$

Solution.

$$\begin{aligned}
\mathcal{L}[\mathbf{y}(t)] &= \mathcal{L}\begin{bmatrix} -e^{-t}\cos 2t + 2e^{-t}\sin 2t \\ 0 \\ 1 + e^{t} \end{bmatrix} \\
&= \begin{bmatrix} -\frac{s+1}{(s+1)^2+4} + \frac{4}{(s+1)^2+4} \\ 0 \\ \frac{1}{s} + \frac{1}{s-1} \end{bmatrix} \\
&= \begin{bmatrix} \frac{3-s}{(s+1)^2+4} \\ 0 \\ \frac{1}{s} + \frac{1}{s-1} \end{bmatrix} \blacksquare
\end{aligned}$$

Problem 49.2
Find $\mathcal{L}[\mathbf{y}(t)]$ where

$$\mathbf{y}(t) = \int_0^t \begin{bmatrix} 1 \\ u \\ e^{-u} \end{bmatrix} du$$

Solution.

$$\mathcal{L}[\mathbf{y}(t)] = \mathcal{L}\begin{bmatrix} t \\ t^2 \\ \end{bmatrix}$$

$$1 \quad 2$$

$$\square \quad e^{-t} + 1$$

$$s^2$$

$$\frac{1}{s} - \frac{1}{s+1} \quad \blacksquare$$

Problem 49.3
Find $\mathcal{L}^{-1}[\mathbf{Y}(s)]$ where

$$\frac{\square}{} \; 1$$

$$s$$

$$\mathbf{Y}(s) = \square \quad \frac{2}{s^2+2s+2}$$

$$\frac{1}{s^2+s}$$

Solution.

We have

$$\begin{aligned}
\mathcal{L}^{-1}\left[\frac{1}{s}\right] &= 1\\
\mathcal{L}^{-1}\left[\frac{2}{s^2+2s+2}\right] &= \mathcal{L}^{-1}\left[\frac{2}{(s+1)^2+1^2}\right] = 2e^{-t}\sin t\\
\mathcal{L}^{-1}\left[\frac{1}{s^2+s}\right] &= \mathcal{L}^{-1}\left[\frac{1}{s}-\frac{1}{s+1}\right] = 1-e^{-t}
\end{aligned}$$

Thus,

$$\mathcal{L}^{-1}[\mathbf{Y}(s)] = \begin{bmatrix} 1 \\ 2e^{-t}\sin t \\ 1-e^{-t} \end{bmatrix}$$

Problem 49.4

Find $\mathcal{L}^{-1}[\mathbf{Y}(s)]$ where

$$\mathbf{Y}(s) = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} \mathcal{L}[t^3] \\ \mathcal{L}[e^{2t}] \\ \mathcal{L}[\sin t] \end{bmatrix}$$

Solution.

We have

$$\begin{aligned}
\mathbf{Y}(s) &= \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} \mathcal{L}[t^3] \\ \mathcal{L}[e^{2t}] \\ \mathcal{L}[\sin t] \end{bmatrix}\\
&= \mathcal{L}\left[\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} t^3 \\ e^{2t} \\ \sin t \end{bmatrix}\right]\\
&= \mathcal{L}\begin{bmatrix} t^3 - e^{2t} + 2\sin t \\ 2t^3 + 3\sin t \\ t^3 - 2e^{2t} + \sin t \end{bmatrix}
\end{aligned}$$

Thus,

$$\begin{bmatrix} t^3 - e^{2t} + 2\sin t \end{bmatrix}$$

$\blacksquare$

Problem 49.5

$$\mathcal{L}^{-1}[\mathbf{Y}(s)] = \begin{bmatrix} 2t^3 + 3\sin t \\ t^3 - 2e^{2t} + \sin t \end{bmatrix}$$

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 5 & -4 \\ 5 & -4 \end{bmatrix}\mathbf{y} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solution.

Taking Laplace of both sides and using the initial condition we find

$$s\mathbf{Y} = \begin{bmatrix} 5 & -4 \\ 5 & -4 \end{bmatrix}\mathbf{Y} + \begin{bmatrix} 0 \\ \frac{1}{s} \end{bmatrix}$$

Solving this matrix equation for $\mathbf{Y}$ we find

$$\mathbf{Y}(s) = \begin{bmatrix} -\frac{4}{s^2(s-1)} \\ \frac{s-5}{s^2(s-1)} \end{bmatrix}.$$

Using partial fractions decomposition we find

$$-\frac{4}{s^2(s-1)} = \frac{4}{s^2} + \frac{4}{s} - \frac{4}{s-1}$$

and

$$\frac{s-5}{s^2(s-1)} = \frac{5}{s^2} + \frac{4}{s} - \frac{4}{s-1}.$$

Hence,

$$\mathbf{Y}(s) = \begin{bmatrix} \frac{4}{s^2} + \frac{4}{s} - \frac{4}{s-1} \\ \frac{5}{s^2} + \frac{4}{s} - \frac{4}{s-1} \end{bmatrix}.$$

Finally,

$$\mathbf{y}(t) = \begin{bmatrix} 4t + 4 - 4e^t \\ 5t + 4 - 4e^t \end{bmatrix} \blacksquare$$

Problem 49.6

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 5 & -4 \end{bmatrix}\mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 3 \end{bmatrix}$$

Solution.

3 −2 2

Taking Laplace of both sides and using the initial condition we find

$$s\mathbf{Y} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix} \mathbf{Y}$$

Solving this matrix equation for $\mathbf{Y}$ we find

$$\mathbf{Y}(s) = \frac{1}{(s-1)(s-2)}\begin{bmatrix} s+2 & -4 \\ 3 & s-5 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$= \frac{1}{(s-1)(s-2)}\begin{bmatrix} 3s-2 \\ 2s-1 \end{bmatrix}$$

Using partial fractions decomposition we find

$$\frac{3s-2}{(s-1)(s-2)} = \frac{-1}{s-1} + \frac{4}{s-2}$$

and

$$\frac{2s-1}{(s-1)(s-2)} = \frac{-1}{s-1} + \frac{3}{s-2}.$$

Hence,

$$\mathbf{Y}(s) = \begin{bmatrix} \frac{-1}{s-1} + \frac{4}{s-2} \\ \frac{-1}{s-1} + \frac{3}{s-2} \end{bmatrix}.$$

Finally,

$$\mathbf{y}(t) = \begin{bmatrix} -e^t + 4e^{2t} \\ -e^t + 3e^{2t} \end{bmatrix} \blacksquare$$

Problem 49.7

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}\mathbf{y} + \begin{bmatrix} 0 \\ 3e^t \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Solution.

Taking Laplace of both sides and using the initial condition we find

$$s\mathbf{Y} - \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}\mathbf{Y} + \begin{bmatrix} 0 \\ \frac{3}{s-1} \end{bmatrix}$$

$$s-1 \quad -4 \qquad 3$$

$s-1$ $\frac{3}{s-1}$

˻

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Solving this matrix equation for $\mathbf{Y}$ we find

$$\mathbf{Y}(s) = \frac{1}{(s-1)^2+4} \begin{matrix} s-1 & 4 \\ -1 & s-1 \end{matrix} \quad \frac{3}{s-1}$$

$$\frac{1}{(s-1)^2+4} \; 3(s-1) + \frac{12}{s-1}$$

Using partial fractions decomposition we find

$$3(s \quad 1) + \frac{12}{s-1} = 3\frac{s-1}{(s-1)^2+4} + \frac{12}{(s-1)[(s-1)^2+4]}.$$

But $\overline{(s-1)^2+4}$

$$\frac{12}{(s-1)[(s-1)^2+4]} = \frac{3}{s-1} - 3\frac{s-1}{(s-1)^2+4}.$$

Hence,

$$\mathbf{Y}(s) = \frac{3}{s-1} \quad .$$

Finally
,

$$\mathbf{y}(t) = \begin{matrix} 0 \\ 3e^t \\ 0 \end{matrix} \quad \blacksquare$$

Problem 49.8

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}'' = \begin{matrix} -3 & -2 \end{matrix} \quad \mathbf{y}, \quad \mathbf{y}(0) = \begin{matrix} 1 \end{matrix}, \quad \mathbf{y}'(0) = \begin{matrix} 0 \end{matrix}$$

Solution.

Taking Laplace of both sides and using the initial condition we find

$$s^2\mathbf{Y} - s \begin{matrix} 1 \end{matrix} - \begin{matrix} 0 \end{matrix} = \begin{matrix} -3 & -2 \end{matrix} \mathbf{Y}$$

$$s^2+3 \quad 2 \qquad\qquad s$$

$$\mathbf{Y} =$$

$$-4 \quad s^2 - 3 \qquad 1$$

Solving this matrix equation for $\mathbf{Y}$ we find

$$\mathbf{Y}(s) = \frac{1}{s^4-1}\left[\begin{array}{cc} s^2-3 & -2 \\ 4 & s^2+3 \end{array}\right]\left[\begin{array}{c} s \\ 1 \end{array}\right] = \frac{1}{s^4-1}\left[\begin{array}{c} s^3-3s-2 \\ s^2+4s+3 \end{array}\right]$$

Using partial fractions decomposition we find

$$\frac{s^3-3s-2}{s^4-1} = -\frac{1}{s-1} + 2\frac{s}{s^2+1} + \frac{1}{s^2+1}$$

and

$$\frac{s^2+4s+3}{(s-1)(s+1)(s^2+1)} = \frac{s+3}{(s-1)(s^2+1)} = \frac{2}{s-1} - \frac{s}{s^2+1} - \frac{1}{s^2+1}.$$

Hence,

$$\mathbf{Y}(s) = \left[\begin{array}{c} -\frac{1}{s-1} + 2\frac{s}{s^2+1} + \frac{1}{s^2+1} \\ \frac{2}{s-1} - \frac{s}{s^2+1} - \frac{1}{s^2+1} \end{array}\right].$$

Finally,

$$\mathbf{y}(t) = \left[\begin{array}{c} -e^t + 2\cos t + \sin t \\ 2e^t - \cos t - \sin t \end{array}\right] \blacksquare$$

Problem 49.9

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}'' = \left[\begin{array}{cc} 1 & -1 \\ 1 & -1 \end{array}\right]\mathbf{y} + \left[\begin{array}{c} 2 \\ 1 \end{array}\right], \quad \mathbf{y}(0) = \left[\begin{array}{c} 0 \\ 1 \end{array}\right], \quad \mathbf{y}'(0) = \left[\begin{array}{c} 0 \\ 0 \end{array}\right]$$

Solution.

Taking Laplace of both sides and using the initial condition we find

$$s^2\mathbf{Y} - s\left[\begin{array}{c} 0 \\ 1 \end{array}\right]$$

$$\frac{2}{}$$

$$1 \qquad\qquad 1 \qquad \frac{1}{s}$$

$$= \quad 1 \;\; -1 \qquad \mathbf{Y} + \;\; s \qquad\qquad -$$

$$s^2 - 1 \qquad 1 \qquad \mathbf{Y} \;=\; \qquad 1 \;\; \frac{2}{s}$$

$$2 \qquad\qquad -\frac{}{s} +$$

$$-1 \qquad s \;\; + 1 \qquad\qquad\qquad s$$

Solving this matrix equation for $\mathbf{Y}$ we find

$$\mathbf{Y}(s) = \frac{1}{s^4}\begin{bmatrix} s^2+1 & -1 \\ 1 & s^2-1 \end{bmatrix}\begin{bmatrix} \frac{2}{s} \\ \frac{1}{s}+s \end{bmatrix} = \begin{bmatrix} \frac{1}{s^3}+\frac{1}{s^5} \\ \frac{1}{s}+\frac{1}{s^5} \end{bmatrix}$$

$$\mathbf{y}(t) = \begin{bmatrix} \frac{t^2}{2}+\frac{t^4}{4!} \\ 1+\frac{t^4}{4!} \end{bmatrix} \quad \blacksquare$$

Problem 49.10

Use the Laplace transform to solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{y} + \begin{bmatrix} e^t \\ 1 \\ -2t \end{bmatrix}, \quad \mathbf{y}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution.

Taking Laplace of both sides and using the initial condition we find

$$s\mathbf{Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{Y} + \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{s} \\ -\frac{2}{s^2} \end{bmatrix}$$

$$\begin{bmatrix} s-1 & 0 & 0 \\ \end{bmatrix} \quad \frac{1}{}$$

$$\begin{bmatrix} s-1 & 0 & 0 \\ 0 & s+1 & -1 \\ 0 & 0 & s-2 \end{bmatrix} \mathbf{Y} = \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{s} \\ -\frac{2}{s^2} \end{bmatrix}$$

Solving this matrix equation for $\mathbf{Y}$ we find

$$\mathbf{Y}(s) = \begin{bmatrix} \frac{1}{s-1} & 0 & 0 \\ 0 & \frac{1}{s+1} & \frac{1}{(s+1)(s-2)} \\ 0 & 0 & \frac{1}{s-2} \end{bmatrix} \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{s} \\ -\frac{2}{s^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{(s-1)^2} \\ \frac{s(s-2)-2}{s^2(s+1)(s-2)} \\ \frac{-2}{s^2(s-2)} \end{bmatrix}$$

Using partial fractions decomposition we find

$$\frac{s(s-2)-2}{s^2(s+1)(s-2)} = \frac{1}{s^2} - \frac{1}{2s} + \frac{1}{3}\cdot\frac{1}{s+1} - \frac{1}{6}\cdot\frac{1}{s-2}$$

and

$$\frac{-2}{s^2(s-2)} = \frac{1}{s^2} + \frac{1}{2s} - \frac{1}{2}\cdot\frac{1}{s-2}.$$

Hence,

$$\mathbf{Y}(s) = \begin{bmatrix} \frac{1}{s^2} - \frac{1}{2s} + \frac{1}{3}\cdot\frac{1}{s+1} - \frac{1}{6}\cdot\frac{1}{s-2} \\ \frac{1}{(s-1)^2} \\ \frac{1}{s^2} + \frac{1}{2s} - \frac{1}{2}\cdot\frac{1}{s-2} \end{bmatrix}.$$

Finally,

$$\mathbf{y}(t) = \begin{bmatrix} t - \frac{1}{2} + \frac{1}{3}e^{-t} - \frac{1}{6}e^{2t} \\ te^t \\ t + \frac{1}{2} - \frac{1}{2}e^{2t} \end{bmatrix}$$

Problem 49.11

The Laplace transform was applied to the initial value problem $\mathbf{y}' = \mathbf{A}\mathbf{y},\ \mathbf{y}(0) = \mathbf{y}_0$, where $\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$, $\mathbf{A}$ is a 2×2 constant matrix, and $\mathbf{y}_0 = \begin{bmatrix} y_{1,0} \\ y_{2,0} \end{bmatrix}$.

The following transform domain solution was obtained

$$\mathcal{L}[\mathbf{y}(t)] = \mathbf{Y}(s) = \frac{1}{s^2 - 9s + 18}\begin{bmatrix} s-2 & -1 \\ 4 & s-7 \end{bmatrix}\begin{bmatrix} y_{1,0} \\ y_{2,0} \end{bmatrix}.$$

(a) what are the eigenvalues of $\mathbf{A}$?

(b) Find $\mathbf{A}$.

Solution.

(a) $det(s\mathbf{I} - \mathbf{A}) = s^2 - 9s + 18 = (s-3)(s-6) = 0$. Hence, the eigenvalues

of $\mathbf{A}$ are $r_1 = 3$ and $r_2 = 6$.

(b) Taking Laplace transform of both sides of the differential equation we find

$$s\mathbf{Y} - \mathbf{y}_0 = \mathbf{A}\mathbf{Y}$$

$$\mathbf{Y} = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}_0$$

Letting $s = 0$ we find

$$\mathbf{Y}(0) = \mathbf{A}^{-1} = \frac{1}{18}\begin{bmatrix} -2 & -1 \\ 4 & -7 \end{bmatrix}$$

Hence,

$$\mathbf{A}^{-1} = \frac{1}{18}\begin{bmatrix} 2 & 1 \\ -4 & 7 \end{bmatrix}$$

and

It follows that

$$det(\mathbf{A}^{-1}) = \frac{1}{18}.$$

$$\mathbf{A} = (\mathbf{A}^{-1})^{-1} = 7 \quad \blacksquare$$

Numerical Differentiationand Integration

Differentiation and integration are basic mathematical operations with a wide range of applications in many areas of science. It is therefore important to have good methods to compute and manipulate derivatives and integrals. You proba- bly learnt the basic rules of differentiation and integration in school — symbolic methods suitable for pencil-and-paper calculations. These are important, and most derivatives can be computed this way. Integration however, is different, and most integrals cannot be determined with symbolic methods like the ones you learnt in school.

Another complication is the fact that in practical applications a function is only known at a few points. For example, we may measure the position of a car every minute via a GPS (Global Positioning System) unit, and we want to compute its speed. If the position is known as a continuous function of time, we can find the speed by differentiating this function. But when the position is only known at isolated times, this is not possible. The same applies to integrals.

The solution, both when it comes to integrals that cannot be determined by the usual methods, and functions that are only known at isolated points, is to use approximate methods of differentiation and integration. In our context, these are going to be numerical methods. We are going to present a number of methods for doing numerical integration and differentiation, but more impor- tantly, we are going to present a general strategy for deriving such methods. In this way you will not only have a number of methods available to you, but you will also be able to develop new methods, tailored to special situations that you may encounter.

We use the same general strategy for deriving both numerical integration and numerical differentiation methods. The basic idea is to evaluate a func- tion at a few points, find the polynomial that interpolates the function at these points, and use the derivative or integral of the polynomial as an approximation to the function. This technique also allows us to keep track of the so-called *trun- cation error*, the mathematical error committed by integrating or differentiating the polynomial instead of the function itself. However, when it comes to round- off error, we have to treat differentiation and integration differently: Numerical integration is very *insensitive* to round-off errors, while numerical differentia- tion behaves in the opposite way; it is very *sensitive* to round-off errors.

A simple method for numerical differentiation

We start by studying numerical differentiation. We first introduce the simplest method, derive its error, and its sensitivity to round-off errors. The procedure used here for deriving the method and analysing the error is used over again in later sections to derive and analyse additional methods.

Let us first make it clear what numerical differentiation is.

Problem 11.1 (Numerical differentiation)**.** *Let f be a given function that is only known at a number of isolated points. The problem of numerical differ- entiation is to compute an approximation to the derivative f' of f by suitablecombinations of the known values of f.*

A typical example is that f is given by a computer program (more specifi- cally a function, procedure or method, depending on you choice of program- ming language), and you can call the program with a floating-point argument x and receive back a floating-point approximation of $f(x)$. The challenge is to compute an approximation to $f'(a)$ for some real number a when the only aid we have at our disposal is the program to compute values of f.

The basic idea

Since we are going to compute derivatives, we must be clear about they are de-fined. Recall that $f'(a)$ is defined by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}. \tag{11.1}$$

In the following we will assume that this limit exists; i.e., that f is differentiable. From (11.1) we immediately have a natural approximation to $f'(a)$; we simply pick a positive h and use the approximation

$$f'(a) \quad \frac{f(a+h)-f(a)}{h}. \tag{11.2}$$

Note that this corresponds to approximating f by the straight line p_1 that inter- polates f at a and $a-h$, and then using $p_1'(a)$ as an approximation to $f'(a)$.

> **Observation 11.2.** *The derivative of f at a can be approximated by*
>
> $$f'(a) \approx \frac{f(a+h)-f(a)}{h}.$$

In a practical situation, the number a would be given, and we would have to locate the two nearest values a_1 and a_2 to the left and right of a such that $f(a_1)$ and $f(a_2)$ can be found. Then we would use the approximation

$$f'(a) \quad \frac{f(a_2)-f(a_1)}{a_2-a_1}.$$

In later sections, we will derive several formulas like (11.2). Which formula to use for a specific example, and exactly how to use it, will have to be decidedin each case.

Example 11.3. Let us test the approximation (11.2) for the function $f(x) = \sin x$ at $a = 0.5$ (using 64-bit floating-point numbers). In this case we have $f'(x) = \cos x$ so $f'(a) = 0.87758256$. This makes it is easy to check the accuracy. We trywith a few values of h and find

h	$\frac{f(a+h)-f(a)}{h}$	$E_1(f;a,h)$
10^{-1}	0.8521693479	2.5×10^{-2}
10^{-2}	0.8751708279	2.4×10^{-3}

10^{-3}	0.8773427029	2.4×10^{-4}
10^{-4}	0.8775585892	2.4×10^{-5}
10^{-5}	0.8775801647	2.4×10^{-6}
10^{-6}	0.8775823222	2.4×10^{-7}

where $E_1(f\,; a, h) = f'(a) - (f(a+h) - f(a))/h$. In other words, the approximation seems to improve with decreasing h, as expected. More precisely, when h is reduced by a factor of 10, the error is reduced by the same factor. ■

The truncation error

Whenever we use approximations, it is important to try and keep track of the error, if at all possible. To analyse the error in numerical differentiation, Taylor polynomials with remainders are useful. To analyse the error in the approxima- tion above, we do a Taylor expansion of $f(a+h)$. We have

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2} f''(\xi_h),$$

where ξ_h lies in the interval $(a, a+h)$. If we rearrange this formula, we obtain

$$f'(a) - \frac{f(a+h) - f(a)}{h} = -\frac{h}{2} f''(\xi_h). \tag{11.3}$$

This is often referred to as the *truncation error* of the approximation, and is a reasonable error formula, but it would be nice to get rid of ξ_h. We first take absolute values in (11.3),

$$\left| f'(a) - \frac{f(a+h) - f(a)}{h} \right| = \frac{h}{2} \left| f''(\xi_h) \right|.$$

Recall from the Extreme value theorem that if a function is continuous, then its maximum always exists on any closed and bounded interval. In our setting here, it is natural to let the closed and bounded interval be $[a, a+h]$. This leads to the following lemma.

Lemma 11.4. *Suppose that f has continuous derivatives up to order two near*

$$\frac{f(a+h) - f(a)}{h},$$

then the truncation error *is bounded*

$$E(f; a, h) = \left| f'(a) - \frac{f(a+h) - f(a)}{h} \right| \le \frac{h}{2} \max_{x \in [a, a+h]} \left| f''(x) \right|. \tag{11.4}$$

Let us check that the error formula (11.3) confirms the numerical values in example 11.3. We have $f''(x) = -\sin x$, so the right-hand side in (11.4) becomes

$$E(\sin; 0.5, h) = \frac{h}{} \sin \xi_h,$$

where $\xi_h \in (0.5, 0.5 + h)$. For $h = 0.1$ we therefore have that the error must lie in the interval

$$[0.05 \sin 0.5, 0.05 \sin 0.6] = [2.397 \times 10^{-2}, 2.823 \times 10^{-2}],$$

and the right end of the interval is the maximum value of the right-hand side in (11.4). When h is reduced by a factor of 10, the factor $h/2$ is reduced by the same factor. This means that ξ_h will approach 0.5 so $\sin \xi_h$ will approach the lower value $\sin 0.5 \approx 0.479426$. For $h = 10^{-n}$, the error will therefore tend to $10^{-n} \sin 0.5/2 \approx 10^{-n} 0.2397$, which is in complete agreement with example 11.3. This is true in general. If f'' is continuous, then ξ_h will approach a when h goes to zero. But even when $h > 0$, the error in using the approximation $f''(\xi_h) \approx f''(a)$ is small. This is the case since it is usually only necessary to know the magnitude of the error, i.e., it is sufficient to know the error with one or two correct digits.

Observation 11.5. *The truncation error is approximately given by*

$$\left| f'(a) - \frac{f(a+h) - f(a)}{h} \right| \approx \frac{h}{2} \left| f''(a) \right|.$$

The round-off error

So far, we have just considered the mathematical error committed when $f'(a)$ is approximated by $\bigl(f(a+h) - f(a)\bigr)/h$. But what about the round-off error? In fact, when we compute this approximation we have to perform the one critical operation $f(a+h) - f(a)$ — subtraction of two almost equal numbers — which we know from chapter 5 may lead to large round-off errors. Let us continue example 11.3 and see what happens if we use smaller values of h.

Example 11.6. Recall that we estimated the derivative of $f(x) = \sin x$ at $a = 0.5$ and that the correct value with ten digits is $f'(0.5) \approx 0.8775825619$. If we check values of h from 10^{-7} and smaller we find

h	$\bigl(f(a+h) - f(a)\bigr)/h$	$E(f; a, h)$
10^{-7}	0.8775825372	2.5×10^{-8}

10^{-8}	0.8775825622	-2.9×10^{-10}
10^{-9}	0.8775825622	-2.9×10^{-10}
10^{-11}	0.8775813409	1.2×10^{-6}
10^{-14}	0.8770761895	5.1×10^{-4}
10^{-15}	0.8881784197	-1.1×10^{-2}
10^{-16}	1.110223025	-2.3×10^{-1}
10^{-17}	0.000000000	8.8×10^{-1}

This shows very clearly that something quite dramatic happens, and when wecome to $h = 10^{-17}$, the derivative is computed as zero.

If $\overline{f(a)}$ is the floating-point number closest to $f(a)$, we know from lemma 5.6 that the relative error will be bounded by 5×2^{-53} since floating-point numbers are represented in binary ($\beta = 2$) with 53 bits for the significand ($m = 53$). We therefore have $|c| \leq 5 \times 2^{-53} \approx 6 \times 10^{-16}$. In practice, the real upper bound on c is usually smaller, and in the following we will denote this upper bound by c^*. This means that a definite upper bound on c^* is 6×10^{-16}.

Notation 11.7. *The maximum relative error when a real number is represented by a floating-point number is denoted by* c^*.

There is a handy way to express the relative error in $f(a)$. If we denote thecomputed value of $f(a)$ by $\overline{f(a)}$, we will have

$$\overline{f(a)} = f(a)(1 + c)$$

which corresponds to the relative error

Observation 11.8. *Suppose that $f(a)$ is computed with 64-bit floating-point numbers and that no underflow or overflow occurs. Then the computed valuef (a) satisfies*

$$\overline{f(a)} = f(a)(1 + c) \tag{11.5}$$

being $|c|$.

The computation of $f(a + h)$ is of course also affected by round-off error, sowe have

$$\overline{f(a)} = f(a)(1 + c_1), \quad \overline{f(a + h)} = f(a + h)(1 + c_2) \tag{11.6}$$

where $|c_i| \leq c^*$ for $i = 1, 2$. Here we should really write $c_2 = c_2(h)$, because the exact round-off error in $f(a + h)$ will inevitably depend on h in a rather random way.

The next step is to see how these errors affect the computed approximation of $f'(a)$. Recall from example 5.11 that the main source of round-off in subtrac- tion is the replacement of the numbers to be subtracted by the nearest floating- point numbers. We therefore consider the computed approximation to be

$$\frac{\overline{f(a+h)}-\overline{f(a)}}{h}.$$

If we insert the expressions (11.6), and also make use of lemma 11.4, we obtain

$$\frac{\overline{f(a+h)}-\overline{f(a)}}{h} = \frac{f(a+h)-f(a)}{h} + \frac{f(a+h)c_2 - f(a)c_1}{h} \tag{11.7}$$

$$= f'(a) + \frac{h}{2} f''(\xi_h) + \frac{f(a+h)c_2 - f(a)c_1}{h}$$

where $\xi_h \in (a, a+h)$. This leads to the following important observation.

Theorem 11.9. *Suppose that f and its first two derivatives are continuous*

$$\frac{\overline{f(a+h)} - \overline{f(a)}}{h},$$

the error in the computed approximation is given by

$$\left| f'(a) - \frac{\overline{f(a+h)} - \overline{f(a)}}{h} \right| \le \frac{h}{2} M_1 + \frac{2c^*}{h} M_2, \tag{11.8}$$

wher

$$M_1 = \max_{x\in[a,a+h]} \left| f''(a) \right|, \quad M_2 = \max_{x\in[a,a+h]} \left| f(a) \right|.$$

Proof. To get to (11.8) we have rearranged (11.7) and used the triangle inequal- ity. We have also replaced $f''(\xi_h)$ by its maximum on the interval $[a, a+h]$, as in (11.4). Similarly, we have replaced $f(\xi_h)$ and $f(a+h)$ by their common maxi- mum on $[a, a+h]$. The last term then follows by applying the triangle inequality to the last term in (11.7) and replacing $|c_1|$ and $|c_2(h)|$ by the upper bound c^*. ∎

The inequality (11.8) can be replaced by an approximate equality by making the approximations $M_1 \approx \left|f''(a)\right|$ and $M_2 \approx \left|f(a)\right|$, just like in observation 11.8 and using the maximum of c_1 and c_2 in (11.7), which we denote $c(h)$.

Observation 11.10. *The inequality* (11.8) *is approxmately equivalent to*

$$\left| f'(a) - \frac{\overline{f(a+h)} - \overline{f(a)}}{h} \right| \approx \frac{h}{2}\left| f''(a) \right| + \frac{2|c(h)|}{h}\left| f(a) \right|. \tag{11.9}$$

Let us check how well observation 11.10 agrees with the computations inexamples 11.3 and 11.6.

Example 11.11. For large values of h the first term on the right in (11.9) will dominate the error and we have already seen that this agrees very well with the computed values in example 11.3. The question is how well the numbers in ex- ample 11.6 can be modelled when h becomes smaller.

To estimate the size of $c(h)$, we consider the case when $h = 10^{-16}$. Then the observed error is -2.3×10^{-1} so we should have

$$\frac{10^{-16}}{2}\sin 0.5 - c\left(10^{-16}\right)\frac{2}{10^{-16}} = -2.3\times10^{-1}.$$

We solve this equation and find

$$c\,10^{-16} \quad \frac{10^{-1}}{6} \quad -1 \quad \frac{10^{-}}{16} \quad 2 \quad \sin \quad -1 \quad 7$$

If we try some other values of h we find

$$c\,10^{-11} = -6.1\times10^{-18},\ c\,10^{-13} = 2.4\times10^{-18},\ c\,10^{-15} = 5.3\times10^{-18}.$$

We observe that all these values are considerably smaller than the upper limit 6×10^{-16} which we mentioned above.

Figure 11.1 shows plots of the error. The numerical approximation has been computed for the values $n = 0.01i$, $i = 0, \ldots, 200$ and plotted in a log-log plot. The errors are shown as isolated dots, and the function

$$g(h) = \frac{h}{2}\sin 0.5 + \tilde{c}\,\frac{2}{h}\sin 0.5 \tag{11.10}$$

with $\tilde{c} = 7\times10^{-17}$ is shown as a solid graph. It seems like this choice of $\tilde{c}$ makes $g(h)$ a reasonable upper bound on the error.

Optimal choice of h

Figure 11.1 indicates that there is an optimal value of h which minimises the total error. We can find this mathematically by

$$g(h) = \frac{h}{\ } \cdot f''(a) \cdot + \frac{2c^*}{\ } \cdot f(a) \cdot . \tag{11.11}$$

minimising the upper bound in (11.9), with $|c(h)|$ replaced by the upper bound c^*. This gives

$$2 \cdot \qquad \cdot \ \frac{\quad}{h} \cdot \qquad \cdot$$

To find the value of h which minimises this expression, we differentiate with respect to h and set the derivative to zero. We find

$$g'(h) = \frac{\cdot f''(a) \cdot}{\quad} - \frac{2c^*}{h^2} \cdot f(a) \cdot .$$

If we solve the equation $g(h) = 0$, we obtain the approximate optimal value.

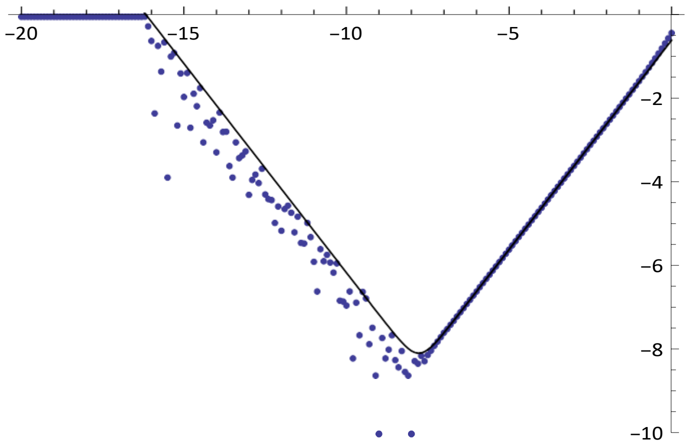


Figure 11.1. Numerical approximation of the derivative of $f(x) = \sin x$ at $x = 0.5$ using the approximation in lemma 11.4. The plot is a $\log_{10}$-$\log_{10}$ plot which shows the logarithm to base 10 of the absolute value of the

total error as a function of the logarithm to base 10 of h, based on 200 values of h. The point -10 on the horizontal axis therefore corresponds $h = 10^{-10}$, and the point -6 on the vertical axis corresponds to an error of 10^{-6}. The plot also includes the function given by (11.10).

Lemma 11.12. *Let f be a function with continuous derivatives up to order 2. If the derivative of f at a is approximated as in lemma 11.4, then the value of h which minimises the total error (truncation error + round-off error) is*

$$h^* \approx 2\sqrt{\frac{\epsilon^* |f(a)|}{|f''(a)|}}.$$

It is easy to see that the optimal value of h is the value that balances the two terms in (11.11)l, i.e., the truncation error and the round-off error are equal. In the example with $f(x) = \sin x$ and a

$= 0.5$ we can use $c^* = 7 \times 10^{-17}$ which gives

$$h^* = 2\sqrt{c^*} = 2\sqrt{7 \times 10^{-17}} \approx 1.7 \times 10^{-8}.$$

Summary of the general strategy

Before we continue, let us sum up the derivaton and analysis of the numericaldifferentiation method in section 11.1, since we will use this over and over again. The first step was to derive the numerical method. In section 11.1 this was very simple since the method came straight out of the definition of the deriva-tive. Just before observation 11.2 we indicated that the method can also be derived by approximating f by a polynomial p and using $p'(a)$ as an approxima- tion to $f'(a)$. This is the general approach that we will use below.

Once the numerical method is known, we estimate the mathematical error in the approximation, *the truncation error*. This we do by performing Taylor expansions with remainders. For numerical differentiation methods which pro- vide estimates of a derivative at a point a, we replace all function values at points other than a by Taylor polynomials with remainders. There may be a challengeto choose the degree of the Taylor polynomial.

The next task is to estimate the total error, including round-off error. We con- sider the difference between the derivative to be computed and the computed approximation, and replace the computed function evaluations by expressions like the ones in observation 11.8. This will result in an expression involving the mathematical approximation to the derivative. This can be simplified in the same way as when the truncation error was estimated, with the addition of an expression involving the relative round-off errors in the function evaluations. These expressions can then be simplified to something like (11.8) or (11.9).

As a final step, the optimal value of h can be found by minimising the totalerror.

Algorithm 11.13. *To derive and analyse a numerical differentiation method, the following steps are necessary:*

1. *Derive the method using polynomial interpolation.*
2. *Estimate the truncation error using Taylor series with remainders.*
3. *Estimate the total error (truncation error + round-off error) by assum- ing all function evaluations are replaced by the nearest floating- pointnumbers.*
4. *Estimate the optimal value of h.*

A simple, symmetric method

The numerical differentiation method in section 11.1 is not symmetric about a, so let us try and derive a symmetric method.

Construction of the method

We want to find an approximation to $f'(a)$ using values of f near a. To obtain a symmetric method, we assume that $f(a-h)$, $f(a)$, and $f(a+h)$ are known values, and we want to find an approximation to $f'(a)$ using these values. The strategy is to determine the quadratic polynomial p_2 that interpolates f at $a-h$, a and $a+h$, and then we use $p_2'(a)$ as an approximation to $f'(a)$.

We write p_2 in Newton form,

$$
\begin{aligned}
p_2(x) = f[a-h] + f[a-h,a](x-(a-h)) \\
+ f[a-h,a,a+h](x-(a-h))(x-a). \qquad (11.12)
\end{aligned}
$$

We differentiate and find

$$p_2'(x) = f[a-h,a] + f[a-h,a,a+h](2x-2a+h).$$

Setting $x = a$ yields

$$p_2'(a) = f[a-h,a] + f[a-h,a,a+h]h.$$

To get a practically useful formula we must express the divided differences in terms of function values. If we expand the second expression we obtain

$$p_2'(a) = f[a-h,a] + \frac{f[a,a+h]-f[a-h,a]}{2h}h = \frac{f[a,a+h]+f[a-h,a]}{2} \qquad (11.13)$$

The two first order differences are

$$f[a-h,a] = \frac{f(a)-f(a-h)}{h}, \quad f[a,a+h] = \frac{f(a+h)-f(a)}{h},$$

and if we insert this in (11.13) we end up with

$$p_2'(a) = \frac{f(a+h)-f(a-h)}{2h}.$$

Lemma 11.14. *Let f be a given function, and let a and h be given numbers. If $f(a-h)$, $f(a)$, $f(a+h)$ are known values, then $f'(a)$ can be approximated by $p_2'(a)$ where p_2 is the quadratic polynomial that interpolates f at $a-h$, a, and $a+h$. The approximation is given by*

$$f'(a) \approx p_2'(a) = \frac{f(a+h)-f(a-h)}{2h}. \qquad (11.14)$$

Let us test this approximation on the function $f(x) = \sin x$ at $a = 0.5$ so wecan compare with the method in section 11.1.

Example 11.15. We test the approximation (11.14) with the same values of h as in examples 11.3 and 11.6. Recall that $f'(0.5) \approx 0.8775825619$ with ten correct decimals. The results are

h	$\big(f(a+h)-f(a-h)\big)/(2h)$	$E(f;a,h)$
10^{-1}	0.8761206554	1.5×10^{-3}
10^{-2}	0.8775679356	1.5×10^{-5}
10^{-3}	0.8775824156	1.5×10^{-7}
10^{-4}	0.8775825604	1.5×10^{-9}
10^{-5}	0.8775825619	1.8×10^{-11}
10^{-6}	0.8775825619	-7.5×10^{-12}
10^{-7}	0.8775825616	2.7×10^{-10}
10^{-8}	0.8775825622	-2.9×10^{-10}
10^{-11}	0.8775813409	1.2×10^{-6}
10^{-13}	0.8776313010	-4.9×10^{-5}
10^{-15}	0.8881784197	-1.1×10^{-2}
10^{-17}	0.0000000000	8.8×10^{-1}

If we compare with examples 11.3 and 11.6, the errors are generally smaller. In particular we note that when h is reduced by a factor of 10, the error is reduced by a factor of 100, at least as long as h is not too small. However, when h be- comes smaller than about 10^{-6}, the error becomes larger. It therefore seems like the truncation error is smaller than in the first method, but the round-off error makes it impossible to get accurate results for small values of h. The optimal value of h seems to be $h^* \approx 10^{-6}$, which is larger than for the first method, but the error is then about 10^{-12}, which is smaller than the best we could do with the first method.

Truncation error

Let us attempt to estimate the truncation error for the method in lemma 11.14. The idea is to do replace $f(a-h)$ and $f(a+h)$ with Taylor expansions about a. We use the Taylor expansions

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2} f''(a)$$

$$f(a-h) = f(a) - hf'(a) + \frac{h^2}{2} f''(a)$$

$(a)+$ $\frac{h^3}{6} f'''(\xi_1)$,

$(a)-$ $\frac{h^3}{6} f'''(\xi_2)$, — f — f

where $\xi_1 \in (a, a + h)$ and $\xi_2 \in (a - h, a)$. If we subtract the second formula from the first and divide by $2h$, we obtain

$$\frac{f(a+h)-f(a-h)}{2h} = f'(a) + \frac{h^2}{12}\left(f'''(\xi_1)+f'''(\xi_2)\right). \qquad (11.15)$$

This leads to the following lemma.

Lemma 11.16. *Suppose that f and its first three derivatives are continuous*

$$\frac{f(a+h)-f(a-h)}{2h}. \tag{11.16}$$

The truncation error in this approximation is bounded by

$$\left|E_2(f;a,h)\right| = \left|f'(a) - \frac{f(a+h)-f(a-h)}{2h}\right| \le \frac{h^2}{6} \max_{x\in[a-h,a+h]} \left|f'''(x)\right|. \tag{11.17}$$

Proof. What remains is to simplify the last term in (11.15) to the term on theright in (11.17). This follows from

$$\begin{aligned}
\left|f'''(\xi_1)+f'''(\xi_2)\right| &\le \max_{x\in[a,a+h]} \left|f'''(x)\right| + \max_{x\in[a-h,a]} \left|f'''(x)\right| \\
&\le \max_{x\in[a-h,a+h]} \left|f'''(x)\right| + \max_{x\in[a-h,a+h]} \left|f'''(x)\right| \\
&= 2 \max_{x\in[a-h,a+h]} \left|f'''(x)\right|.
\end{aligned}$$

The last inequality is true because the width of the intervals over which we take the maximums are increased, so the maximum values may also increase.

The error formula (11.17) confirms the numerical behaviour we saw in ex- ample 11.15 for small values of h since the error is proportional to h^2: When h is reduced by a factor of 10, the error is reduced by a factor 10^2.

Round-off error

The round-off error may be estimated just like for the first method. When the ap- proximation (11.16) is computed, the values $f(a-h)$ and $f(a+h)$ are replaced by the nearest floating point numbers $f(a$

$-\, h$) and $f\,(a + h)$ which can be ex- pressed as

$$\overline{f\,(a+h)} = f\,(a+h)(1+c_1),\quad \overline{f\,(a-h)} = f\,(a-h)(1+c_2),$$

where both c_1 and c_2 depend on h and satisfy $|c_i| \le c^*$ for $i = 1, 2$. Using these expressions we obtain

$$\frac{\overline{f\,(a+h)} - \overline{f\,(a-h)}}{2h} = \frac{f\,(a+h) - f\,(a-h)}{2h} + \frac{f\,(a+h)c_1 - f\,(a-h)c_2}{2h}.$$

We insert (11.15) and get the relation

$$\frac{f(a+h)-f(a-h)}{2h} = f'(a) + \frac{h^2}{12}\big(f'''(\xi_1)+f'''(\xi_2)\big) + \frac{f(a+h)c_1 - f(a-h)c_2}{2h}.$$

This leads to an estimate of the total error if we use the same technique as in the proof of lemma 11.8.

Theorem 11.17. *Let f be a given function with continuous derivatives up to order three, and let a and h be given numbers. Then the error in the approxi-*

$$f'(a) \approx \frac{f(a+h)-f(a-h)}{2h},$$

including round-off error and truncation error, is bounded by

$$\left| f'(a) - \frac{\overline{f(a+h)} - \overline{f(a-h)}}{2h} \right| \le \frac{h^2}{6} M_1 + \frac{c^*}{h} M_2 \tag{11.18}$$

wher

$$M_1 = \max_{x\in[a-h,a+h]} |f'''(x)|, \quad M_2 = \max_{x\in[a-h,a+h]} |f(x)|. \tag{11.19}$$

In practice, the interesting values of h will usually be so small that there is very little error in making the approximations

$$M_1 = \max_{x\in[a-h,a+h]} |f'''(x)| \approx |f'''(a)|, \qquad M_2 = \max_{x\in[a-h,a+h]} |f(x)| \approx |f(a)|.$$

If we make this simplification in (11.18) we obtain a slightly simpler error esti-mate.

Observation 11.18. *The error* (11.18) *is approximately bounded by*

$$\left| f'(a) - \frac{\overline{f(a+h)} - \overline{f(a-h)}}{2h} \right| \lesssim \frac{h^2}{6} |f'''(a)| + \frac{c^* |f(a)|}{h}. \tag{11.20}$$

A plot of how the error behaves in this approximation, together with the es- timate of the error on the right in (11.20), is shown in figure 11.2.

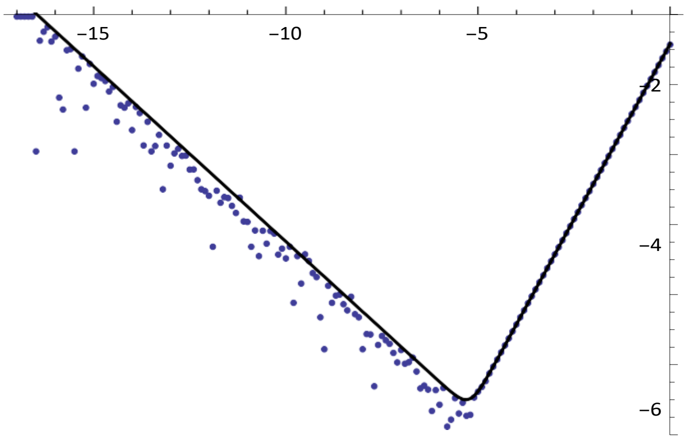


Figure 11.2. Log-log plot of the error in the approximation to the derivative of $f(x) = \sin x$ at $x = 1/2$ for values of h in the interval $[0, 10^{-17}]$, using the method in theorem 11.17. The function plotted is the right-hand side of (11.20) with $c^* = 7 \times 10^{-17}$, as a function of h.

Optimal choice of h

As for the first numerical differentiation method, we can find an optimal value of h which minimises the error. The error is minimised when the truncation error and the round-off error have the same magnitude. We can find this value of h if we differentiate the right-hand side of (11.18) with respect to h and set the derivative to 0. This leads to the equation

$$\frac{h}{3}M_1 - \frac{c^*}{h^2}M_2 = 0$$

which has the solution

$$\sqrt[3]{3c^* M_2} \qquad \sqrt[3]{3c^* \cdot f(a) \cdot}$$

$$h^* = \frac{}{3\,\overline{M1}} \approx q. \quad 3 \quad \cdot f'''(a)\cdot$$

At the end of section 11.1.4 we saw that a reasonable value for c^* was $c^* = 7\times 10^{-17}$. The optimal value of h in example 11.15, where $f(x) = \sin x$ and $a = 1/2$, then becomes $h = 4.6 \times 10^{-6}$. For this value of h the approximation is $f'(0.5) \approx 0.877582561887$ with error 3.1×10^{-12}.

A four-point method for differentiation

In a way, the two methods for numerical differentiation that we have consid- ered so far are the same. If we use a step length of $2h$ in the first method, the approximation becomes

$$f'(a) \approx \frac{f(a+2h)-f(a)}{2h}.$$

The analysis of the symmetric method shows that the approximation is consid- erably better if we associate the approximation with the midpoint between a and $a+h$,

$$f'(a+h) \approx \frac{f(a+2h)-f(a)}{2h}.$$

At the point $a+h$ the approximation is proportional to h^2 rather than h, and this makes a big difference as to how quickly the error goes to zero, as is evident if we compare examples 11.3 and 11.15. In this section we derive another method for which the truncation error is proportional to h^4.

The computations below may seem overwhelming, and have in fact been done with the help of a computer to save time and reduce the risk of miscal- culations. The method is included here just to illustrate that the principle for deriving both the method and the error terms is just the same as for the simple symmetric method in the previous section. To save space we have only included one highlight, of the approximation method and the total error.

Derivation of the method

We want better accuracy than the symmetric method which was based on inter- polation with a quadratic polynomial. It is therefore natural to base the approxi- mation on a cubic polynomial, which can interpolate four points. We have seen the advantage of symmetry, so we choose the interpolation points $x_0 = a-2h$, $x_1 = a-h$, $x_2 = a+h$, and $x_3 = a+2h$. The cubic polynomial that interpolates f at these points is

$$p_3(x) = f(x_0) + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$
$$+ f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2).$$

and its derivative is

$$p_3'(x) = f[x_0, x_1] + f[x_0, x_1, x_2](2x - x_0 - x_1)$$

$$+ f[x_0, x_1, x_2, x_3] \ (x - x_1)(x - x_2) + (x - x_0)(x - x_2) + (x - x_0)(x - x_1) \ .$$

If we evaluate this expression at $x = a$ and simplify (this is quite a bit of work), we find that the resulting approximation of $f'(a)$ is

$$f'(a) \approx p_3'(a) = \frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{12h}. \quad (11.21)$$

Truncation error

To estimate the error, we expand the four terms in the numerator in (11.21) in Taylor series,

$$f(a-2h) = f(a) - 2hf'(a) + 2h^2 f''(a) - \frac{4h^3}{3} f'''(a) + \frac{2h^4}{3} f^{(iv)}(a) - \frac{4h^5}{15} f^{(v)}(\xi_1),$$

$$f(a-h) = f(a) - hf'(a) + \frac{h^2}{2} f''(a) - \frac{h^3}{6} f'''(a) + \frac{h^4}{24} f^{(iv)}(a) - \frac{h^5}{120} f^{(v)}(\xi_2),$$

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2} f''(a) + \frac{h^3}{6} f'''(a) + \frac{h^4}{24} f^{(iv)}(a) + \frac{h^5}{120} f^{(v)}(\xi_3),$$

$$f(a+2h) = f(a) + 2hf'(a) + 2h^2 f''(a) + \frac{4h^3}{3} f'''(a) + \frac{2h^4}{3} f^{(iv)}(a) + \frac{4h^5}{15} f^{(v)}(\xi_4),$$

where $\xi_1 \in (a-2h, a)$, $\xi_2 \in (a-h, a)$, $\xi_3 \in (a, a+h)$, and $\xi_4 \in (a, a+2h)$. If we insert this into the formula for $p_3'(a)$ we obtain

$$\frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{12h} = f'(a) - \frac{h^4}{45} f^{(v)}(\xi_1) + \frac{h^4}{180} f^{(v)}(\xi_2) + \frac{h^4}{180} f^{(v)}(\xi_3) - \frac{h^4}{45} f^{(v)}(\xi_4).$$

If we use the same trick as for the symmetric method, we can combine all last four terms in and obtain an upper bound on the truncation error. The result is

$$\left| f'(a) - \frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{\quad} \right| \quad h^4$$

$$f'(a) - \frac{\quad}{12h} \leq \frac{\quad}{18} M \qquad (11.22)$$

where $M = \max_{x\in[a-2h,a+2h]} \left| f^{(v)}(x) \right|$.

Round-off error

The truncation error is derived in the same way as before. The quantities we actually compute are

$$\overline{f(a-2h)} = f(a-2h)(1+c_1), \quad \overline{f(a+2h)} = f(a+2h)(1+c_3),$$

$$\overline{f(a-h)} = f(a-h)(1+c_2), \quad \overline{f(a+h)} = f(a+h)(1+c_4).$$

We estimate the difference between $f'(a)$ and the computed approximation, make use of the estimate (11.22), combine the function values that are multi- plied by cs, and approximate the maximum values by function values at a. Wesum up the result.

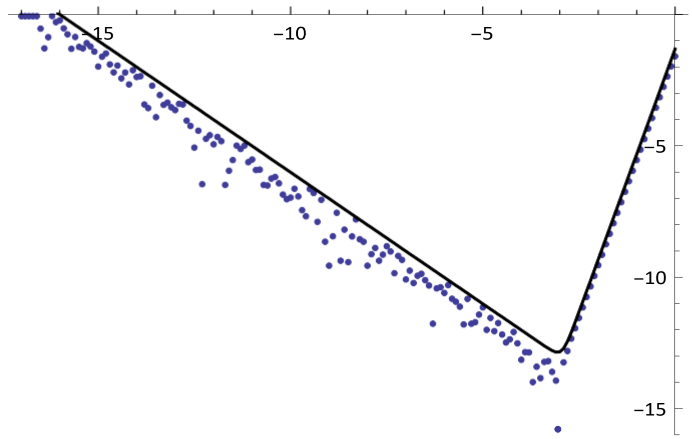


Figure 11.3. Log-log plot of the error in the approximation to the derivative of $f(x) = \sin x$ at $x = 1/2$, using the method in observation 11.19, with h in the interval $[0, 10^{-17}]$. The function plotted is the right-hand side of (11.23) with $c^* = 7 \times 10^{-17}$.

Observation 11.19. *Suppose that f and its first five derivatives are continu-*

$$f'(a) \approx \frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{12h},$$

the total error is approximately bounded by

$$\left| f'(a) - \frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{12h} \right| \lesssim \frac{h^4}{18} \left| f^{(v)}(a) \right| + \frac{3c^*}{h} \left| f(a) \right|. \quad (11.23)$$

A plot of the error in the approximation for the $\sin x$ example is shown infigure 11.3.

Optimal value of *h*

From observation 11.19 we can compute the optimal value of h by differentiat- ing the right-hand side with respect to h and setting it to zero,

$$\frac{2h^3}{9}\cdot f^{(v)}(a)\cdot - \frac{3c^*}{h^2}\cdot f(a)\cdot = 0$$

which has the solution

$$h^* = \sqrt[5]{\frac{27c^* \cdot |f(a)|}{f^{(v)}}}$$

For the case above with $f(x) = \sin x$ and $a = 0.5$ the solution is $h^* \approx 8.8 \times 10^{-4}$. For this value of h the actual error is 10^{-14}.

Numerical approximation of the second derivative

We consider one more method for numerical approximation of derivatives, this time of the second derivative. The approach is the same: We approximate f by a polynomial and approximate the second derivative of f by the second derivative of the polynomial. As in the other cases, the error analysis is based on expansion in Taylor series.

Derivation of the method

Since we are going to find an approximation to the second derivative, we have to approximate f by a polynomial of degree at least two, otherwise the second derivative is identically 0. The simplest is therefore to use a quadratic polyno- mial, and for symmetry we want it to interpolate f at $a - h$, a, and $a + h$. The resulting polynomial p_2 is the one we used in section 11.3 and it is given in equa- tion (11.12). The second derivative of p_2 is constant, and the approximation of $f'(a)$ is

$$f''(a) \approx p_2''(a) = f[a-h, a, a+h].$$

The divided difference is easy to expand.

> **Lemma 11.20.** *The second derivative of a function f at a can be approxi-*
>
> $$f''(a) \approx \frac{f(a+h) - 2f(a) + f(a-h)}{h^2}. \qquad (11.24)$$

The truncation error

Estimation of the error goes as in the other cases. The Taylor series of $f(a-h)$ and $f(a+h)$ are

$$f(a-h) = f(a) - hf'(a) + \frac{h^2}{2}f''(a) - \frac{h^3}{6}f'''(a) + \frac{h^4}{24}f^{(iv)}(\xi_1),$$

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2}f''(a) + \frac{h^3}{6}f'''(a) + \frac{h^4}{24}f^{(iv)}(\xi_2),$$

where $\xi_1 \in (a-h, a)$ and $\xi_2 \in (a, a+h)$. If we insert these Taylor series in (11.24)we obtain

$$\frac{f(a+h)-2f(a)+f(a-h)}{h^2} = f''(a) + \frac{h^2}{\;}\big(f^{(iv)}(\xi_1)+f^{(iv)}(\xi_2)\big).$$

From this obtain an expression for the truncation error.

Lemma 11.21. *Suppose f and its first three derivatives are continuous near*

$$f''(a) \approx \frac{f(a+h)-2f(a)+f(a-h)}{h^2},$$

the error is bounded by

$$\left| f''(a) - \frac{f(a+h)-2f(a)+f(a-h)}{h^2} \right| \le \frac{h^2}{12} \max_{x\in[a-h,a+h]} \left| f'''(x) \right|. \qquad (11.25)$$

Round-off error

The round-off error can also be estimated as before. Instead of computing the exact values, we compute $\overline{f(a-h)}$, $\overline{f(a)}$, and $\overline{f(a+h)}$, which are linked to the exact values by

$$\overline{f(a-h)} = f(a-h)(1+c_1),\quad \overline{f(a)} = f(a)(1+c_2),\quad \overline{f(a+h)} = f(a+h)(1+c_3),$$

where $|c_i| \le c^*$ for $i = 1, 2, 3$. The difference between $f''(a)$ and the computed approximation is therefore

$$f''(a) - \frac{\overline{f(a+h)}-2\overline{f(a)}+\overline{f(a-h)}}{h^2} = -\frac{h^2}{24}\big(f'''(\xi_1)+f'''(\xi_2)\big) - \frac{c_1 f(a-h) - c_2 f(a) + c_3 f(a+h)}{h^2}.$$

If we combine terms on the right as before, we end up with the following theo-rem.

Theorem 11.22. *Suppose f and its first three derivatives are continuous near*

$$f''(a) \approx \frac{f(a+h) - 2f(a) + f(a-h)}{h^2}.$$

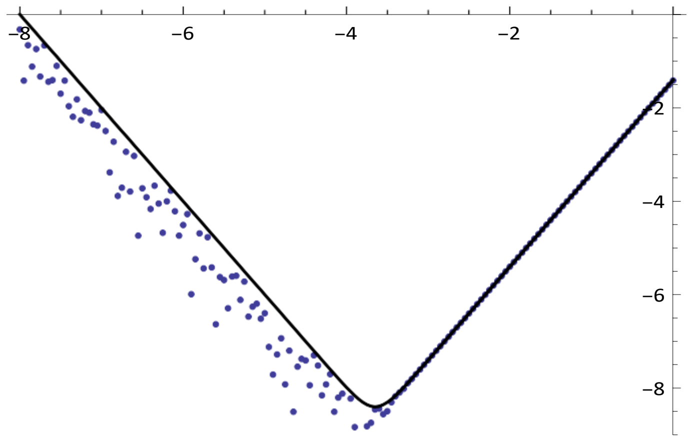


Figure 11.4. Log-log plot of the error in the approximation to the derivative of $f(x) = \sin x$ at $x = 1/2$ for h in the interval $[0, 10^{-8}]$, using the method in theorem 11.22. The function plotted is the right-hand side of (11.23) with $c^* = 7 \times 10^{-17}$.

Then the total error (truncation error + round-off error) in the computed ap-

$$\left| f''(a) - \frac{\overline{f(a+h)} - 2\overline{f(a)} + \overline{f(a-h)}}{h^2} \right| \le \frac{h^2}{12} M_1 + \frac{3c^*}{h^2} M_2. \qquad (11.26)$$

wher

$$M_1 = \max_{x \in [a-h, a+h]} \left| f^{(iv)}(x) \right|, \quad M_2 = \max_{x \in [a-h, a+h]} \left| f(x) \right|.$$

As before, we can simplify the right-hand side to

$$\frac{h^2}{12} \left| f^{(iv)}(a) \right| + \frac{3c^*}{h^2} \left| f(a) \right| \qquad (11.27)$$

if we can tolerate a slightly approximate upper bound.

Figure 11.4 shows the errors in the approximation to the second derivative given in theorem 11.22 when $f(x) = \sin x$ and a

= 0.5 and for h in the range $[0, 10^{-8}]$. The solid graph gives the function in (11.27) which describes the upper limit on the error as function of h, with $c^* = 7 \times 10^{-17}$. For h smaller than 10^{-8}, the approximation becomes 0, and the error constant. Recall that for the ap- proximations to the first derivative, this did not happen until h was about 10^{-17}. This illustrates the fact that the higher the derivative, the more problematic is the round-off error, and the more difficult it is to approximate the derivative with numerical methods like the ones we study here.

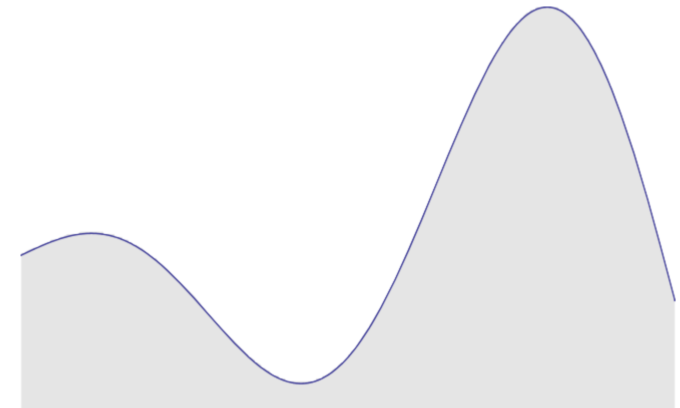


Figure 11.5. The area under the graph of a function.

Optimal value of *h*

Again, we find the optimal value of h by minimising the right-hand side of (11.26). To do this we find the derivative with respect to h and set it to 0,

$$\frac{h}{6}M_1 - \frac{6c^*}{h^3}M_2 = 0.$$

As usual it does not make much difference if we use the approximations M_1 $|f'''(a)|$ and $M_2 = |f(a)|$.

> **Observation 11.23.** *The upper bound on the total error* (11.26) *is minimised when h has the*
>
> $$h^* = \sqrt[4]{\frac{36c^*|f(a)|}{|f^{(iv)}(a)|}}.$$

When $f(x) = \sin x$ and $a = 0.5$ this gives $h^* = 2.2 \times 10^{-4}$ if we use the value $c^* = 7 \times 10^{-17}$. Then the approximation to $f''(a) = -\sin a$ is

-0.4794255352 with an actual error of 3.4×10^{-9}.

General background on integration

Our next task is to develop methods for numerical integration. Before we turn to this, it is necessary to briefly review the definition of the integral.

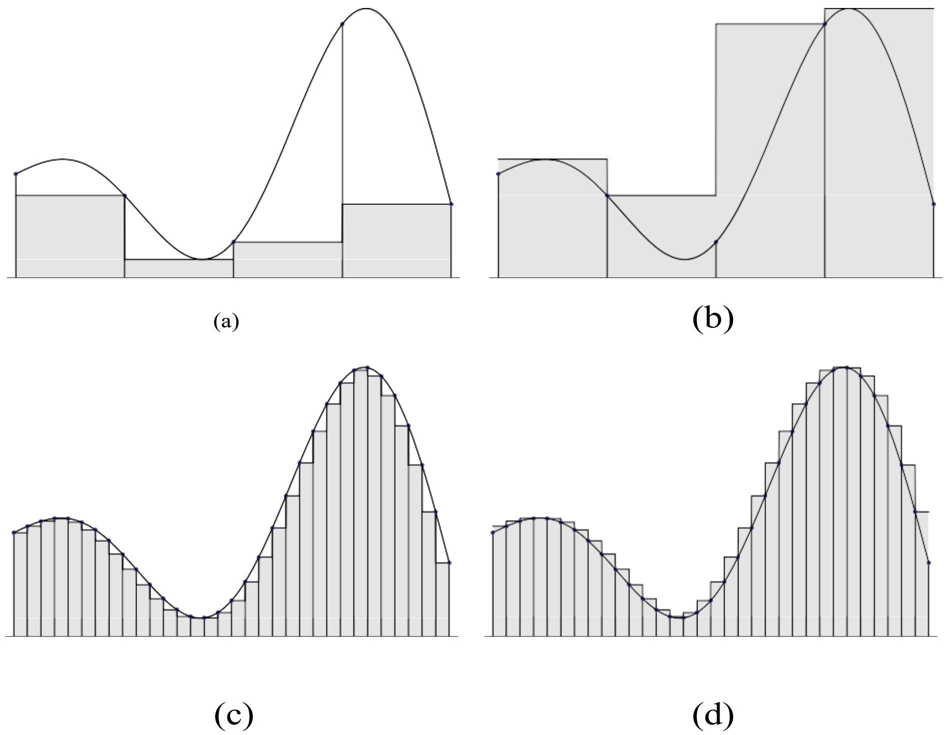


Figure 11.6. The definition of the integral via inscribed and circumsribed step functions.

Recall that if $f(x)$ is a function, then the integral of f from $x = a$ to $x = b$ iswritten

$$\int_a^b f(x)\,dx.$$

This integral gives the area under the graph of f, with the area under the positive part counting as positive area, and the area under the negative part of f counting as negative area, see figure 11.5.

Before we continue, we need to define a term which we will use repeatedly in our description of integration.

Definition 11.24. *Let a and b be two real numbers with $a < b$. A* partition *of $[a, b]$ is a finite sequence $\{x_i\}^n_{i=0}$ of increasing numbers in $[a, b]$ with $x_0 = a$ and $x_n = b$,*

$$a = x_0 < x_1 < x_2 \cdots < x_{n-1} < x_n = b.$$

The partition is said to be uniform *if there is a fixed number h, called the* step length*, such that $x_i - x_{i-1} = h = (b - a)/n$ for $i = 1, \ldots, n$.*

The traditional definition of the integral is based on a numerical approxi- mation to the area. We pick a partition $\{x_i\}$ of $[a, b]$, and in each subinterval $[x_{i-1}, x_i]$ we determine the maximum and minimum of f (for convenience we assume that these values exist),

$$m_i = \min_{x\in[x_{i-1}, x_i]} f(x), \quad M_i = \max_{x\in[x_{i-1}, x_i]} f(x),$$

for $i = 1, 2, \ldots, n$. We use these values to compute the two sums

$$\underline{I} = \sum_{i=1}^{n} m_i (x_i - x_{i-1}), \quad \overline{I} = \sum_{i=1}^{n} M_i (x_i - x_{i-1}).$$

To define the integral, we consider larger partitions and consider the limits of $\underline{I}$ and $\overline{I}$ as the distance between neighbouring x_i s goes to zero. If those limits are the same, we say that f is integrable, and the integral is given by this limit. More precisely,

$$I = \int_a^b f(x)\,dx = \sup \underline{I} = \inf \overline{I},$$

where the sup and inf are taken over all partitions of the interval $[a, b]$. This process is illustrated in figure 11.6 where we see how the piecewise constant approximations become better when the rectangles become narrower.

The above definition can be used as a numerical method for computing ap- proximations to the integral. We choose to work with either maxima or minima, select a partition of $[a, b]$ as in figure 11.6, and add together the areas of the rect- angles. The problem with this technique is that it can be both difficult and time consuming to determine the maxima or minima, even on a computer.

However, it can be shown that the integral has a very convenient property: If we choose a point t_i in each interval $[x_{i-1}, x_i]$, then the sum

$$\tilde{I} = \sum_{i=1}^{n} f(t_i)(x_i - x_{i-1})$$

will also converge to the integral when the distance between neighbouring x_i s goes to zero. If we choose t_i equal to x_{i-1} or x_i, we have a simple numerical method for computing the integral. An even better choice is the more symmetric $t_i = (x_i + x_{i-1})/2$ which leads to the approximation

$$I \approx \sum_{i=1}^{n} f\big((x_i + x_{i-1})/2\big)(x_i - x_{i-1}). \tag{11.28}$$

This is the so-called *midpoint method* which we will study in the next section.

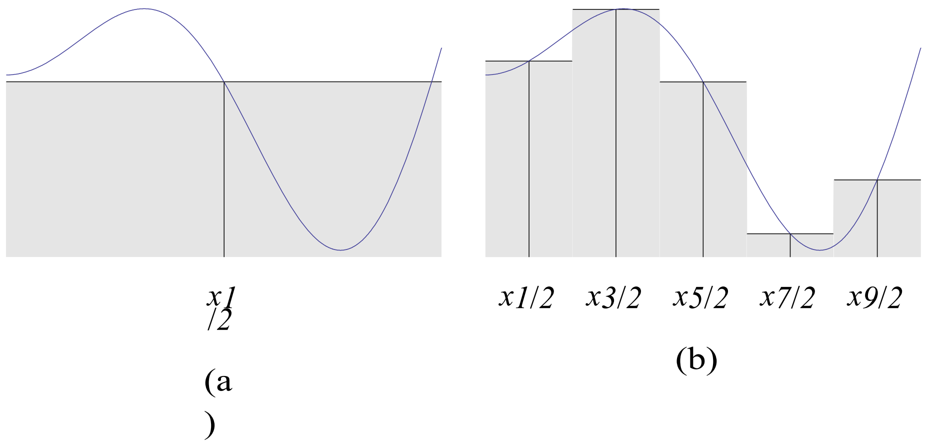


Figure 11.7. The midpoint rule with one subinterval (a) and five subintervals (b).

In general, we can derive numerical integration methods by splitting the interval $[a, b]$ into small subintervals, approximate f by a polynomial on each subinterval, integrate this polynomial rather than f, and then add together the contributions from each subinterval. This is the strategy we will follow, and this works as long as f can be approximated well by polynomials on each subinter-val.

The midpoint method for numerical integration

We have already introduced the midpoint rule (11.28) for numerical integration. In our standard framework for numerical methods based on polynomial approx- imation, we can consider this as using a constant approximation to the function f on each subinterval. Note that in the following we will always assume the par- tition to be uniform.

Algorithm 11.25. *Let f a function which is integrable on the interval $[a,b]$*

[illegible]

gral of f is approximated by

$$\int_a^b f(x)\,dx \approx I_{mid}(h) = h\sum_{i=1}^{n} f(x_{i-1/2}), \tag{11.29}$$

wher

$$x_{i-1/2} = (x_i + x_{i-1})/2 = a + (i - 1/2)h.$$

This may seem like a strangely formulated algorithm, but all there is to it is to compute the sum on the right in (11.29). The method is illustrated in figure 11.7 in the cases where we have one and five subintervals.

Example 11.26. Let us try the midpoint method on an example. As usual, it is wise to test on an example where we know the answer, so we can easily check the quality of the method. We choose the integral

$$\int_0^1 \cos x \, dx = \sin 1 \approx 0.8414709848$$

where the exact answer is easy to compute by traditional, symbolic methods. To test the method, we split the interval into 2^k subintervals, for $k = 1, 2, \ldots, 10$, i.e., we halve the step length each time. The result is

h	$I_{mid}(h)$	Error
0.500000	0.85030065	-8.8×10^{-3}
0.250000	0.84366632	-2.2×10^{-3}
0.125000	0.84201907	-5.5×10^{-4}
0.062500	0.84160796	-1.4×10^{-4}
0.031250	0.84150523	-3.4×10^{-5}
0.015625	0.84147954	-8.6×10^{-6}
0.007813	0.84147312	-2.1×10^{-6}
0.003906	0.84147152	-5.3×10^{-7}
0.001953	0.84147112	-1.3×10^{-7}
0.000977	0.84147102	-3.3×10^{-8}

By error, we here mean

$$\int_0^1 f(x)\,dx - I_{mid}(h).$$

Note that each time the step length is halved, the error seems to be reduced by a factor of 4.

Local error analysis

As usual, we should try and keep track of the error. We first focus on what hap- pens on one subinterval. In other words we want to study the error

$$\int_a^b f(x)\,dx$$

$$f(a_{1/2})(b-a), \quad a_{1/2} = (a+b)/2. \tag{11.30}$$

Once again, Taylor polynomials with remainders help us out. We expand both $f(x)$ and $f(a_{1/2})$ about the the left endpoint

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2} f''(\xi_1),$$

$$f(a_{1/2}) = f(a) + (a_{1/2}-a)f'(a) + \frac{(a_{1/2}-a)^2}{2} f''(\xi_2),$$

where ξ_1 is a number in the interval (a, x) that depends on x, and ξ_2 is in the interval $(a, a_{1/2})$. If we multiply the second Taylor expansion by $(b-a)$, we obtain

$$f(a_{1/2})(b-a) = f(a)(b-a) + \frac{(b-a)^2}{2} f'(a) + \frac{(b-a)^3}{8} f''(\xi_2). \tag{11.31}$$

Next, we integrate the Taylor expansion and obtain

$$\begin{aligned}\int_a^b f(x)\,dx &= \int_a^b \left(f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2} f''(\xi_1) \right) dx \\ &= f(a)(b-a) + \left[\frac{(x-a)^2}{2} \right]_a^b f'(a) + \frac{1}{2}\int_a^b (x-a)^2 f''(\xi_1)\,dx \\ &= f(a)(b-a) + \frac{(b-a)^2}{2} f'(a) + \frac{1}{2}\int_a^b (x-a)^2 f''(\xi_1)\,dx. \end{aligned} \tag{11.32}$$

We then see that the error can be written

$$\left| \int_a^b f(x)\,dx - f(a_{1/2})(b-a) \right| = \left| \frac{1}{2}\int_a^b (x-a)^2 f''(\xi_1)\,dx - \frac{(b-a)^3}{8} f''(\xi_2) \right| \le \frac{1}{2}\left| \int_a^b (x-a)^2 f''(\xi_1)\,dx \right| + \frac{(b-a)^3}{8} \left| f''(\xi_2) \right|.$$

For the last term, we use our standard trick,

$$\left| f''(\xi_2) \right| \le M = \max_{x\in[a,b]} \left| f''(x) \right|. \tag{11.34}$$

Note that since $\xi_2 \in (a, a_{1/2})$, we could just have taken the maximum over the interval $[a, a_{1/2}]$, but we will see later that it is more convenient to maximise over the whole interval $[a, b]$.

The first term in (11.33) needs some massaging. Let us do the work first, and explain afterwords,

$$
\begin{aligned}
\left|\frac{1}{2}\int_a^b (x-a)^2 f''(\xi_1)\,dx\right| &\le \frac{1}{2}\int_a^b \left|(x-a)^2 f''(\xi_1)\right| dx \\
&\le \frac{M}{2}\int_a^b (x-a)^2\,dx \\
&= \frac{M}{2}\left[\frac{(x-a)^3}{3}\right]_a^b \\
&= \frac{M}{6}(b-a)^3 .
\end{aligned}
\tag{11.35}
$$

The first inequality is valid because when we move the absolute value sign in- side the integral sign, the function that we integrate becomes nonnegative ev- erywhere. This means that in the areas where the integrand in the original ex- pression is negative, everything is now positive, and hence the second integral is larger than the first.

Next there is an equality which is valid because $(x-a)^2$ is never negative. The next inequality follows because we replace $f''(\xi_1)$ with its maximum on the interval $[a, b]$. The last step is just the evaluation of the integral of $(x-a)^2$.

We have now simplified both terms on the right in (11.33), so we have

$$\left| \int_a^b f(x)\,dx - f\big(a_{1/2}\big)(b-a) \right| \le \frac{M}{6}(b-a)^3 + M(b-a)^3.$$

The result is the

Lemma 11.27. *Let f be a continuous function whose first two derivatives are continuous on the interval $[a.b]$. The the error in the midpoint method, with only one interval, is bounded by*

$$\left| \int_a^b f(x)\,dx - f\big(a_{1/2}\big)(b-a) \right| \le \frac{7M}{24}(b-a)^3,$$

where $M = \max_{x\in[a,b]} |f''(x)|$ and $a_{1/2} = (a+b)/2$.

The importance of this lemma lies in the factor $(b-a)^3$. This means that if we reduce the size of the interval to half its width, the error in the midpoint method will be reduced by a factor of 8.

Perhaps you feel completely lost in the work that led up to lemma 11.27. The wise way to read something like this is to first focus on the general idea that was used: Consider the error (11.30) and replace both $f(x)$ and $f(a_{1/2})$ by its quadratic Taylor polynomials with remainders. If we do this, a number of terms cancel out and we are left with (11.33). At this point we use some standardtechniques that give us the final inequality.

Once you have an overview of the derivation, you should check that the de- tails are correct and make sure you understand each step.

Global error analysis

Above, we analysed the error on one subinterval. Now we want to see what hap- pens when we add together the contributions from many subintervals; it should not surprise us that this may affect the error.

We consider the general case where we have a partition that divides $[a, b]$ into n subintervals, each of width h. On each subinterval we use the simple midpoint rule that we analysed in the previous section,

$$I = \int_a^b f(x)\,dx = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} f(x)\,dx \approx \sum_{i=1}^{n} f(x_{i-1/2})h.$$

The total error is then

$$I - I_{mid} = \sum_{i=1}^{n} \left(\int_{x_{i-1}}^{x_i} f(x)\,dx - f(x_{i-1/2})h \right).$$

But the expression inside the parenthesis is just the local error on the interval $[x_{i-1}, x_i]$. We therefore have

$$|I - I_{mid}| = \left| \sum_{i=1}^{n} \left(\int_{x_{i-1}}^{x_i} f(x)\,dx - f(x_{i-1/2})h \right) \right|$$

$$\le \sum_{i=1}^{n} \left| \int_{x_{i-1}}^{x_i} f(x)\,dx - f(x_{i-1/2})h \right|$$

$$\le \sum_{i=1}^{n} \frac{h^3}{24} M_i \tag{11.36}$$

where M_i is the maximum of $|f''(x)|$ on the interval $[x_{i-1}, x_i]$. To simiplify the expression (11.36), we extend the maximum on $[x_{i-1}, x_i]$ to all of $[a, b]$. This will usually make the maximum larger, so for all i we have

$$M_i = \max_{x \in [x_{i-1}, x_i]} |f''(x)| \le \max_{x \in [a,b]} |f''(x)| = M.$$

Now we can simplify (11.36) further,

$n \quad 3 \quad n \quad 3 \quad 3$

$$\sum_{i=1} \frac{7h}{} M_i \le \sum_{i=1} \frac{7h}{} M = \frac{7h}{} nM. \qquad (11.37)$$

Here, we need one final little observation. Recall that $h = (b-a)/n$, so $hn = b-a$. If we insert this in (11.37), we obtain our main result.

Theorem 11.28. *Suppose that f and its first two derivatives are continuouson the interval $[a,b]$, and that the integral of f on $[a,b]$ is approximated by*

$$I = \int_a^b f(x)\,dx \approx I_{mid} = \sum_{i=1}^{n} f(x_{i-1/2})h.$$

Then the error is bounded by

$$|I - I_{mid}| \leq (b-a)\,\frac{7h^2}{24}\max_{x\in[a,b]} \left|f''(x)\right| \tag{11.38}$$

where $x_{i-1/2} = a + (i - 1/2)h$.

This confirms the error behaviour that we saw in example 11.26: If h is re- duced by a factor of 2, the error is reduced by a factor of $2^2 = 4$.

One notable omission in our discussion of the midpoint method is round-off error, which was a major concern in our study of numerical differentiation. The good news is that round-off error is not usually a problem in numerical integra- tion. The only situation where round-off may cause problems is when the value of the integral is 0. In such a situation we may potentially add many numbers that sum to 0, and this may lead to cancellation effects. However, this is so rarethat we will not discuss it here.

You should be aware of the fact that the error estimate (11.38) is not the best possible in that the constant 7/24 can be reduced to 1/24, but then the deriva- tion becomes much more complicated.

Estimating the step length

The error estimate (11.38) lets us play a standard game: If someone demands that we compute an integral with error smaller than c, we can find a step length h that guarantees that we meet this demand. To make sure that the error is smaller than c, we enforce the inequality

$$(b-a)\,\frac{7h^2}{24}\max_{x\in[a,b]} f''(x) \quad c$$

which we can easily solve for h,

$$h \leq \sqrt{\frac{24c}{7(b-a)M}}, \qquad M = \max_{x\in[a,b]} |f''(x)|.$$

This is not quite as simple as it may look since we will have to estimate M, the maximum value of the second derivative. This can be difficult, but in some casesit is certainly possible, see exercise 4.

A detailed algorithm

Algorithm 11.25 describes the midpoint method, but lacks a lot of detail. In thissection we give a more detailed algorithm.

Whenever we compute a quantity numerically, we should try and estimate the error, otherwise we have no idea of the quality of our computation. A stan- dard way to do this for numerical integration is to compute the integral for de- creasing step lengths, and stop the computations when difference between two successive approximations is less than the tolerance. More precisely, we choose an initial step length h_0 and compute the approximations

$$I_{mid}(h_0), I_{mid}(h_1), \ldots, I_{mid}(h_k), \ldots,$$

where $h_k = h_0/2^k$. Suppose $I_{mid}(h_k)$ is our latest approximation. Then we esti-mate the relative error by the number

$$\frac{|I_{mid}(h_k) - I_{mid}(h_{k-1})|}{|I_{mid}(h_k)|}$$

and stop the computations if this is smaller than c. To avoid potential divisionby zero, we use the test

$$|I_{mid}(h_k) - I_{mid}(h_{k-1})| \leq c|I_{mid}(h_k)|.$$

As always, we should also limit the number of approximations that are com-puted.

Algorithm 11.29. *Suppose the function f, the interval $[a,b]$, the length n_0 the intitial partition, a positive tolerance $c < 1$, and the maximum number iterations M are given. The following algorithm will compute a sequence of approximations to $\int_a^b f(x)dx$ by the midpoint rule, until the estimated rela-*

tive error is smaller than c, or the maximum number of computed approxi-mations reach M. The final approximation is stored in I.

$n := n_0$; $h := (b - a)/n$;

$I := 0$; $x := a + h/2$;

for $k := 1, 2, \ldots, n$ I
  $:= I + f(x)$;

  $x := x + h$;

$j := 1$;

$I := h * I$;

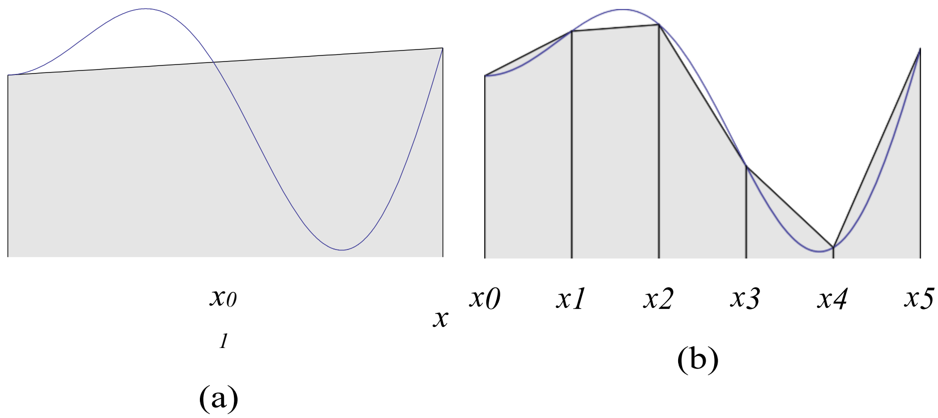


Figure 11.8. The trapezoid rule with one subinterval (a) and five subintervals (b).

```
n := 2n;   h := (b − a)/n;

I := 0;   x := a + h/2;

for k := 1, 2, . . . , nI
       := I + f (x);

       x := x + h;
```

Note that we compute the first approximation outside the main loop. This is necessary in order to have meaningful estimates of the relative error (the first time we reach the top of the while loop we will always get past the condition). We store the previous approximation in *Ip* so that we can estimate the error.

In the coming sections we will describe two other methods for numerical integration. These can be implemented in algorithms similar to Algorithm 11.29. In fact, the only difference will be how the actual approximation to the integral is computed.

The trapezoid rule

The midpoint method is based on a very simple polynomial approximation to the function f to be integrated on each subinterval; we simply use a constant approximation by interpolating the function value at the middle point. We are now going to consider a natural alternative; we approximate f on each subin- terval with the secant that interpolates f at both ends of the subinterval.

The situation is shown in figure 11.8a. The approximation to the integral is the area of the trapezoidal figure under the secant so we have

$$\int_a^b f(x)\,dx \approx \frac{f(a)+f(b)}{2}(b-a). \tag{11.39}$$

To get good accuracy, we will have to split $[a, b]$ into subintervals with a partition and use this approximation on each subinterval, see figure 11.8b. If we have a uniform partition $\{x_i\}_{i=0}^n$ with step length h, we get the approximation

$$\int_a^b f(x)\,dx = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} f(x)\,dx \approx \sum_{i=1}^{n} \frac{f(x_{i-1})+f(x_i)}{2} h. \tag{11.40}$$

We should always aim to make our computational methods as efficient as pos- sible, and in this case an improvement is possible. Note that on the interval $[x_{i-1}, x_i]$ we use the function values $f(x_{i-1})$ and $f(x_i)$, and on the next inter- val we use the values $f(x_i)$ and $f(x_{i+1})$. All function values, except the first and last, therefore occur twice in the sum on the right in (11.40). This means that if we implement this formula directly we do a lot of unnecessary work. From the explanation above the following observation follows.

Observation 11.30 (Trapezoid rule). *Suppose we have a function f definedon an interval $[a,b]$ and a partition $\{x_i\}_{i=0}^n$ of $[a.b]$. If we approximate f secant on each subinterval and approximate the integral of f by the of the resulting piecewise linear approximation, we obtain the*

$$\int_a^b f(x)\,dx \approx h\left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f(x_i)\right). \tag{11.41}$$

In the formula (11.41) there are no redundant function evaluations.

Local error analysis

Our next step is to analyse the error in the trapezoid method. We follow the same recipe as for the midpoint method and use Taylor series. Because of the similarities with the midpoint method, we will skip some of the details.

We first study the error in the approximation (11.39) where we only have one secant. In this case the error is given by

$$\int_a^b f(x)\,dx - \frac{f(a)+f(b)}{2}(b-a), \tag{11.42}$$

and the first step is to expand the function values $f(x)$ and $f(b)$ in Taylor series about a,

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2} f''(\xi_1),$$
$$f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2} f''(\xi_2),$$

where $\xi_1 \in (a, x)$ and $\xi_2 \in (a, b)$. The integration of the Taylor series for $f(x)$ we did in (11.32) so we just quote the result here,

$$\int_a^b f(x)\,dx = f(a)(b-a) + \frac{(b-a)^2}{2} f'(a) + \frac{1}{2}\int_a^b (x-a)^2 f''(\xi_1)\,dx.$$

If we insert the Taylor series for $f(b)$ we obtain

$$\frac{f(a)+f(b)}{2}(b-a) = f(a)(b-a) + \frac{(b-a)^2}{2} f'(a) + \frac{(b-a)^3}{4} f''(\xi_2).$$

If we insert these expressions into the error (11.42), the first two terms cancel against each other, and we obtain

$$\left|\int_a^b f(x)\,dx - \frac{f(a)+f(b)}{2}(b-a)\right| \le \left|\frac{1}{2}\int_a^b (x-a)^2 f''(\xi_1)\,dx - \frac{(b-a)^3}{4} f''(\xi_2)\right|.$$

These expressions can be simplified just like in (11.34) and (11.35), and this yields

$$\left|\int_a^b f(x)\,dx - \frac{f(a)+f(b)}{2}(b-a)\right| \le \frac{M}{6}(b-a)^3 + \frac{M}{4}(b-a)^3.$$

Let us sum this

Lemma 11.31. *Let f be a continuous function whose first two derivatives are continuous on the interval $[a.b]$. The the error in the trapezoid rule, with only*

$$\left| \int_a^b f(x)\,dx - \frac{f(a)+f(b)}{2}(b-a) \right| \leq \frac{5M}{12}(b-a)^3,$$

where $M = \max_{x\in[a,b]} \left| f''(x) \right|$.

This lemma is completely analogous to lemma 11.27 which describes the local error in the midpoint method. We particularly notice that even though the trapezoid rule uses two values of f, the error estimate is slightly larger than the estimate for the midpoint method. The most important feature is the exponent on $(b-a)$, which tells us how quickly the error goes to 0 when the interval width is reduced, and from this point of view the two methods are the same. In other words, we have gained nothing by approximating f by a linear functions instead of a constant. This does not mean that the trapezoid rule is bad, it rather means that the midpoint rule is unusually good.

Global error

We can find an expression for the global error in the trapezoid rule in exactly the same way as we did for the midpoint rule, so we skip the proof. We sum everything up in a theorem about the trapezoid rule.

Theorem 11.32. *Suppose that f and its first two derivatives are continuous*

$$I = \int_a^b f(x)\,dx \approx I_{trap} = h\left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f(x_i)\right).$$

Then the error is bounded by

$$\left|I - I_{trap}\right| \le (b-a)\,\frac{5h^2}{12}\max_{x\in[a,b]}\left|f''(x)\right|. \tag{11.43}$$

As we mentioned when we commented on the midpoint rule, the error es- timates that we obtain are not best possible in the sense that it is possible to derive better error estimates (using other techniques) with smaller constants. In the case of the trapezoid rule, the constant can be reduced from 5/12 to 1/12. However, the fact remains that the trapezoid rule is a disappointing method compared to the midpoint rule.

Simpson's rule

The final method for numerical integration that we consider is *Simpson's rule*. This method is based on approximating f by a parabola on each subinterval, which makes the derivation a bit more involved. The error analysis is essentially the same as before, but because the expressions are more complicated, it pays off to plan the analysis better. You may therefore find the material in this sec- tion more challenging than the treatment of the other two methods, and should make sure that you have a good understanding of the error analysis for these methods before you start studying section 11.9.2.

Deriving Simpson's rule

As for the other methods, we derive Simpson's rule in the simplest case where we use one parabola on all of $[a, b]$. We find the polynomial p_2 that interpolates f at a, $a_{1/2} = (a + b)/2$ and b, and approximate the integral of f by the integral of p_2. We could find p_2 via the Newton form, but in this case it is easier to use the Lagrange form. Another simiplification is to first contstruct Simpson's rule in the case where $a = -1$, $a_{1/2} = 0$, and $b = 1$, and then use this to generalise the method.

The Lagrange form of the polyomial that interpolates f at $-1, 0, 1$, is given by

$$p_2(x) = f(-1)\frac{x(x-1)}{2} - f(0)(x+1)(x-1) + f(1)\frac{(x+1)x}{2},$$

and it is easy to check that the interpolation conditions hold. To integrate p_2, we must integrate each of the three polynomials in this expression. For the first one we have

$$\frac{1}{2}\int_{-1}^{1} x(x-1)\,dx = \frac{1}{2}\int_{-1}^{1} (x^2 - x)\,dx = \frac{1}{2}\Big[\frac{1}{3}x^3 - \frac{1}{2}x^2\Big]_{-1}^{1} = \frac{1}{3}.$$

Similarly, we find

$$-\int_{-1}^{1} (x+1)(x-1)\,dx = \frac{4}{3}, \quad \frac{1}{2}\int_{-1}^{1} (x+1)x\,dx = \frac{1}{3}.$$

On the interval $[-1, 1]$, Simpson's rule therefore gives the approximation

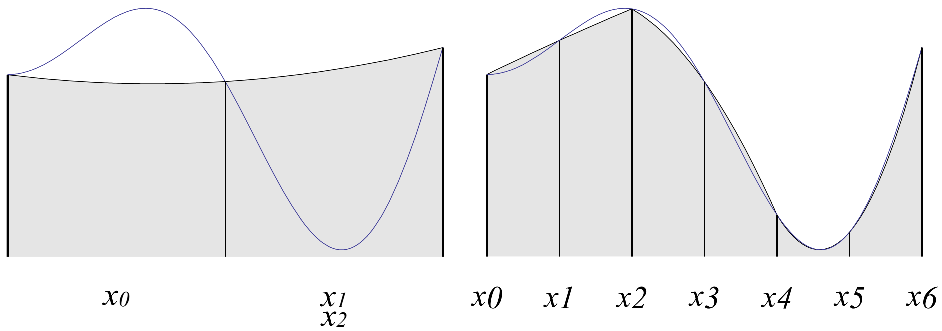
$$\int_{-1}^{1} f(x)\,dx \approx \frac{1}{3}\big(f(-1) + 4f(0) + f(1)\big). \tag{11.44}$$

To obtain an approximation on the interval $[a, b]$, we use a standard tech-nique. Suppose that x and y are related by

$$x = (b-a)\frac{y+1}{2} + a. \tag{11.45}$$

We see that if y varies in the interval $[-1, 1]$, then x will vary in the interval $[a, b]$. We are going to use the relation (11.45) as a substitution in an integral, so we note that $dx = (b-a)dy/2$. We therefore have

$$\int_a^b f(x)\,dx = \int_{-1}^{1} f\left((b-a)\frac{y+1}{2} + a\right)\frac{b-a}{2}\,dy = \frac{b-a}{2}\int_{-1}^{1} \tilde{f}(y)\,dy, \tag{11.46}$$



(a) (b)

Figure 11.9. Simpson's rule with one subinterval (a) and three subintervals (b).

where

$$\tilde{f}(y) = f\left(\frac{b-a}{2}(y+1)+a\right).$$

To determine an approximaiton to the intgeral of $\tilde{f}$ on the interval $[-1, 1]$, we can use Simpson's rule (11.44). The result is

$$\int_{-1}^{1} \tilde{f}(y)\,dy \approx \frac{1}{3}\left(\tilde{f}(-1) + 4\tilde{f}(0) + \tilde{f}(1)\right) = \frac{1}{3}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right),$$

since the relation in (11.45) maps -1 to a, the midpoint 0 to $(a + b)/2$, and the right endpoint b to 1. If we insert this in (11.46), we obtain Simpson's rule for the general interval $[a, b]$, see figure 11.9a. In practice, we will usually divide the interval $[a, b]$ into smaller ntervals and use Simpson's rule on each subinterval, see figure 11.9b.

Observation 11.33. *Let f be an integrable function on the interval $[a, b]$. If f is interpolated by a quadratic polynomial p_2 at the points a, $(a+b)/2$ and b, then the integral of f can be approximated by the integral of*

$$\int_a^b f(x)\,dx \approx \int_a^b p_2(x)\,dx = \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right). \tag{11.47}$$

We could just as well have derived this formula by doing the interpolation directly on the interval $[a, b]$, but then the algebra becomes quite messy.

Local error analysis

The next step is to analyse the error. We follow the usual recipe and perform Tay- lor expansions of $f(x)$, $f(a+b)/2$ and $f(b)$ around the left endpoint a. How- ever, those Taylor expansions become long and tedious, so we are gong to see how we can predict what happens. For this, we define the error function,

$$E(f) = \int_a^b f(x)\,dx - \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right). \tag{11.48}$$

Note that if $f(x)$ is a polynomial of degree 2, then the interpolant p_2 will be ex- actly the same as f. Since the last term in (11.48) is the integral of p_2, we see that the error $E(f)$ will be 0 for any quadratic polynomial. We can check this by calculating the error for the three functions 1, x, and x^2,

$$E(1) = (b-a) - \frac{b-a}{6}(1+4+1) = 0,$$

$$E(x) = \frac{1}{2}\Big[x^2\Big]_a^b - \frac{b-a}{6}\left(a + 4\frac{a+b}{2} + b\right) = \frac{1}{2}(b^2-a^2) - \frac{(b-a)(b+a)}{2} = 0,$$

$$\begin{aligned} E(x^2) &= \frac{1}{3}\Big[x^3\Big]_a^b - \frac{b-a}{6}\left(a^2 + 4\frac{(a+b)^2}{4} + b^2\right) \\ &= \frac{1}{3}(b^3-a^3) - \frac{b-a}{3}(a^2+ab+b^2) \\ &= \frac{1}{3}\left(b^3-a^3-(a^2b+ab^2+b^3-a^3-a^2b-ab^2)\right) \\ &= 0. \end{aligned}$$

Let us also check what happens if $f(x) = x^3$,

$$\begin{aligned} E(x^3) &= \frac{1}{4}\Big[x^4\Big]_a^b - \frac{b-a}{6}\left(a^3 + 4\frac{(a+b)^3}{8} + b^3\right) \\ &= \frac{1}{4}(b^4-a^4) - \frac{b-a}{6}\left(a^3 + \right. \end{aligned}$$

$$\frac{a^3+3a^3b+3ab^2+b^3}{3}$$

$$= \frac{1}{4}(b^4-a^4) - \frac{b-a}{6}\times\frac{3}{2}\left(a^3+3a^2b+3ab^2+b^3\right)$$

$$=0.$$

The fact that the error is zero when $f(x) = x^3$ comes as a pleasant surprise; by the construction of the method we can only expect the error to be 0 for quadratic polynomials.

The above computations mean that

$$E\left(c_0+c_1x+c_2x^2+c_3x^3\right) = c_0E(1)+c_1E(x)+c_2E(x^2)+c_3E(x^3) = 0$$

for any real numbers $\{c_i\}_{i=0}^{3}$, i.e., the error is 0 whenever f is a cubic polynomial.

Lemma 11.34. *Simpson's rule is exact for cubic polynomials.*

To obtain an error estimate, suppose that f is a general function which can be expanded in a Taylor polynomial of degree 3 about a with remainder

$$f(x) = T_3(f;x) + R_3(f;x).$$

Then we see that

$$E(f) = E\big(T_3(f) + R_3(f)\big) = E\big(T_3(f)\big) + E\big(R_3(f)\big) = E\big(R_3(f)\big).$$

The second equality follows from simple properties of the integral and function evaluations, while the last equality follows because the error in Simpson's rule is 0 for cubic polynomials.

The Lagrange form of the error term is given by

$$R_3(f;x) = \frac{(x-a)^4}{24} f^{(iv)}(\xi_x),$$

where $\xi_x \in (a,x)$. We then find

$$E\big(R_3(f;x)\big) = \frac{1}{24}\int_a^b (x-a)^4 f^{(iv)}(\xi_x)\,dx - \frac{b-a}{6}\left(0 + 4\frac{(b-a)^4}{24\times 16} f^{(iv)}(\xi_{(a+b)/2}) + \frac{(b-a)^4}{24} f^{(iv)}(\xi_b)\right)$$

$$= \frac{1}{24}\int_a^b (x-a)^4 f^{(iv)}(\xi_x)\,dx - f^{(iv)} \; \frac{(b-a)^5}{}$$

(iv) (iv) $(\xi_{(a+b)/2}) + 4f$ (ξ_b) ,

where $\xi_1 = \xi_{(a+b)/2}$ and $\xi_2 = \xi_b$ (the error is 0 at $x = a$). If we take absolute values, use the triangle inequality, the standard trick of replacing the function values by maxima over the whole interval $[a,b]$, and evaluate the integral, we obtain

$$|E(f)| \leq \frac{(b \quad a)^5}{5 \times 24} M + \frac{(b \quad a)^5}{576}(M + 4M).$$

This gives the following estimate of the error.

Lemma 11.35. *If f is continuous and has continuous derivatives up to order*

$$|E(f)| < \frac{49}{}(b-a)^5 \max_{x\in[a,b]} .f^{(iv)}(x)..$$

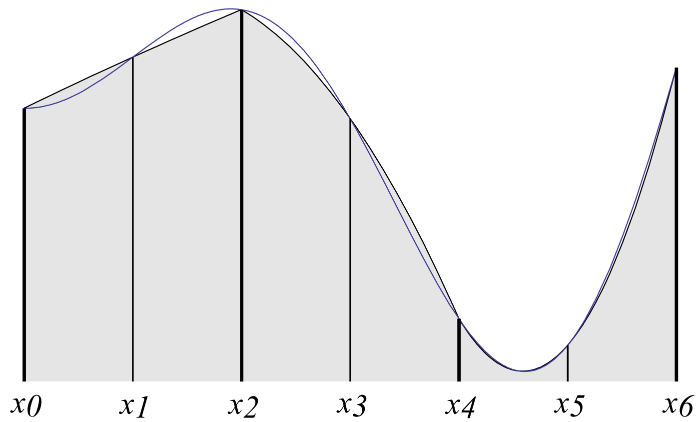


Figure 11.10. Simpson's rule with three subintervals.

We note that the error in Simpson's rule depends on $(b-a)^5$, while the error in the midpoint rule and trapezoid rule depend on $(b-a)^3$. This means that the error in Simpson's rule goes to zero much more quickly than for the other two methods when the width of the interval $[a, b]$ is reduced. More precisely, a reduction of h by a factor of 2 will reduce the error by a factor of 32.

As for the other two methods the constant 49/2880 is not best possible; it canbe reduced to 1/2880 by using other techniques.

Composite Simpson's rule

Simpson's rule is used just like the other numerical integration techniques we have studied: The interval over which f is to be integrated is split into subin- tervals, and Simpson's rule is applied on neighbouring pairs of intervals, see fig- ure 11.10. In other words, each parabola is defined over *two* subintervals which means that the total number of subintervals must be even and the number of given values of f must be odd.

If the partition is $(x_i)^{2n}$ with $x_i = a + ih$, Simpson's rule on the interval

$[x_{2i-2}, x_{2i}]$ is

$$\int_{x_{2i-2}}^{x_{2i}} f(x)\,dx \approx \frac{h}{3}\left(f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})\right).$$

The approximation to the total integral is therefore

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\sum_{i=1} (f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})).$$

In this sum we observe that the right endpoint of one subinterval becomes the left endpoint of the following subinterval to the right. Therefore, if this is im- plemented directly, the function values at the points with an even subscript will be evaluated twice, except for the extreme endpoints a and b which only occur once in the sum. We can therefore rewrite the sum in a way that avoids these redundant evaluations.

Observation 11.36. *Suppose f is a function defined on the interval $[a,b]$, and let $\{x_i\}_{i=0}^{2n}$ be a uniform partition of $[a,b]$ with step length h. The composite Simpson's rule approximates the integral of f by*

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f(a)+f(b)+2\sum_{i=1}^{n-1} f(x_{2i})+4\sum_{i=1}^{n} f(x_{2i-1})\right).$$

With the midpoint rule, we computed a sequence of approximations to the integral by successively halving the width of the subintervals. The same is often done with Simpson's rule, but then care should be taken to avoid unnecessary function evaluations since all the function values computed at one step will alsobe used at the next step, see exercise 7.

The error in the composite Simpson's rule

The approach we used to deduce the global error for the midpoint rule, can also be used for Simpson's rule, see theorem 11.28. The following theorem sums this up.

Theorem 11.37. *Suppose that f and its first four derivatives are continuous on the interval $[a, b]$, and that the integral of f on $[a, b]$ is approximated by Simpson's rule with $2n$ subintervals of equal width h. Then the error is*

$$\left|E(f)\right| \le (b-a)\,\frac{49h^4}{2880}\max_{x\in[a,b]}\left|f^{(iv)}(x)\right|. \tag{11.49}$$

Summary

In this chapter we have derived a three methods for numerical differentiation and three methods for numerical integration. All these methods and their error analyses may seem rather overwhelming, but they all follow a common thread:

Procedure 11.38. *The following is a general procedure for deriving numerical methods for differentiation and integration:*

1. *Interpolate the function f by a polynomial p at suitable points.*
2. *Approximate the derivative or integral of f by the derivative or integral of p. This makes it possible to express the approximation in terms of function values of f.*
3. *Derive an estimate for the error by expanding the function values (otherthan the one at a) in Taylor series with remainders.*

4D. For numerical differentiation, derive an estimate of the round-off error by assuming that the relative errors in the function values are bounded by ϵ^*. By minimising the total error, an optimal step length h can be determined.

4I. For numerical integration, the global error can easily be derived from the local error using the technique leading up to theorem 11.28

Perhaps the most delicate part of the above procedure is to choose the de- gree of the Taylor polynomials. This is discussed in exercise 6.

It is procedure 11.38 that is the main content of this chapter. The individual methods are important in practice, but also serve as examples of how this proce- dure is implemented, and should show you how to derive other methods more suitable for your specific needs.

Exercises

a) Write a program that implements the numerical differentiation method

$$f'(a) \approx \frac{f(a+h) - f(a-h)}{2h},$$

and test the method on the function $f(x) = e^x$ at $a = 1$.

b) Determine the optimal value of h given in section 11.3.4 which minimises the total error. Use $c^* = 7 \times 10^{-17}$.

c) Use your program to determine the optimal value h of experimentally.

d) Use the optimal value of h that you found in (c) to determine a better value for c^* in this specific example.

Repeat exercise 1, but compute the second derivative using the approximation

$$f''(a) \approx \frac{f(a+h)-2f(a)+f(a-h)}{h^2}.$$

In (b) you should use the value of h given in observation 11.23.

a) Suppose that we want to derive a method for approximating the derivative of f at a which has the form

$$f'(a) \approx c_1 f(a-h) + c_2 f(a+h), \quad c_1, c_2 \in \mathbf{R}.$$

We want the method to be exact when $f(x) = 1$ and $f(x) = x$. Use these conditions to determine c_1 and c_2.

b) Show that the method in (a) is exact for all polynomials of degree 1, and compare it to the methods we have discussed in this chapter.

c) Use the procedure in (a) and (b) to derive a method for approximating the second derivative of f,

$$f''(a) \approx c_1 f(a-h) + c_2 f(a) + c_3 f(a+h),$$

$c_1, c_2, c_3 \in \mathbf{R}$, by requiring that the method should be exact when $f(x) = 1$, x and x^2.

d) Show that the method in (c) is exact for all quadratic polynomials.

a) Write a program that implements the midpoint method as in algorithm 11.29 and test it on the integral

$$\int_0^1 e^x\,dx = e - 1.$$

b) Determine a value of h that guarantees that the absolute error is smaller than 10^{-10}. Run your program and check what the actual error is for this value of h. (You may have to adjust algorithm 11.29 slightly and print the absolute error.)

Repeat exercise 4, but use Simpson's rule instead of the midpoint method.

It may sometimes be difficult to judge how many terms to include in the Taylor series used in the analysis of numerical methods. In this exercise we are going to see how this can be done. We use the numerical approximation

$$f'(a) \approx \frac{f(a+h)-f(a-h)}{2h}$$

in section 11.3 for our experiments.

a) Do the same derivation as section 11.3.2, but include only two terms in the Taylor series (plus remainder). What happens?

b) Do the same derivation as section 11.3.2, but include four terms in the Taylor series (plus remainder). What happens now?

When h is halved in the trapezoid method, some of the function values used with step length $h/2$ are the same as those used for step length h. Derive a formula for the trapezoid method with step length $h/2$ that makes use of the function values that were computed for step length h.

(12) *ASSIGNMENT QUESTION/ INNOVATIVE ASSIGNMENTSETS*

Unit-I

1. Find the root of the equation x log x =1.2 using false position method.

2. Find the root of the equation x^3-x-3=0

3. By the method of least squares find the straight line that best fits the following data

x	1	2	3	4	5
y	14	27	40	55	68

4. Fit a parabola to the following data using the method of least squares.

x	1	3	7	9	11	13
y	3.49	8.69	19.09	24.29	29.49	34.69

5. Fit a polynomial of second degree to the data points (2, 3.07), (4, 12.85),(6, 31.47), (8, 57.38) and (10, 91.29).

6. Fit the curve y= ae^{bx} to the following data.

x	0	0.5	1.0	1.5	2.0	2.5
y	0.10	0.45	2.15	9.15	40.35	180.7 5

7. Using Newton –Raphson method find square root of 24
8. Derive normal equations of a parabola by method of least squares

Unit-II

1. Dividing the range into 10 equal parts , find the value of $\int_0^{\frac{\pi}{2}} \sin x \, dx$, using i) . Trapezoidal rule

ii). Simpson's $\frac{1}{3}$ rd rule. iii) Simpson's $\frac{3}{8}$ th rule

2. Use R-K method to evaluate y(0.1) and y(0.2) given that y' = x + y , y(0) = 1.

3. Use Taylors series method to find the approximate value of y when x=0.1 given that y(0)=1 ,y'=3x+y^2.

4. Find y(0.1)&y(0.2) using Euler's modified form given that y'=x^2-y, y(0)=1.

Unit-III

1. Define derivative of a complex function and find derivative of $f(z) = z^n$ where n is +ve integer.
2. If $u = e^x(xcosy - ysiny)$ then find analytic function of f(z).
3. Show that $u = \frac{x}{x^2+y^2}$ is harmonic.
4. State and prove Cauchy-Riemann equations in polar form.
5. Show that $f(z) = z + \bar{z}$ is not analytic any where in the complex plane.
6. Define analytic function and entire function.
7. Find whether $f(z) = \frac{x-iy}{x^2+y^2}$ is not analytic or not.
8. Find all values of "k" such that $f(z) = e^x(xcosky + i\, ysinky)$ is analytic.
9. Prove that z^n (n is a +ve integer) is analytic and hence find its derivative.
10. Find analytic function whose real part is $\frac{x}{x^2+y^2}$.
11. Prove that the function of $f(z)$ defined by $f(z) = \begin{cases} \frac{x^3(1+x)-y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

is continuous and C-R equations at the origin ,yet $f'(0)$ does not exist.

12. If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$.

13 . Determine "p" so that the function $f(z) = \frac{1}{2}\log(x^2 + y^2) + tan^{-1}(\frac{px}{y})$ is analytic.

14. Find the analytic function $f(z) = u + iv$ if $u - v = e^x(cosy - siny)$.

15. Determine the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 = 1$. Also find the harmonic conjugate of this real part.

16.Discuss the continuity of $f(x,y) = \begin{cases} \frac{2xy(x+y)}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$

17. Construct the analytic function $f(z)$, whose real part is $e^x cosy$.

18. If $u(x,y) and\ v(x,y)$ are harmonic functions in a region R, prove that the function $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is an analytic function.

19. Prove that $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$ is not analytic at $z = 0$ although C-R equations are satisfied at origin.

20. Prove that $f(z) = \sqrt{|xy|}$ is not analytic at $z = 0$ although C-R equations are satisfied at origin.

21. State and prove Cauchy-Riemann equations in Cartesian coordinates.

Unit-IV

1. Expand $f(z) = sinz$ in Taylor's series about $z = \frac{\pi}{4}$.
2. Find Residue of $f(z) = \frac{z}{z^2+1}$ at its poles.
3. Find Residues of $f(z) = \frac{1}{(z+1)(z+2)}$ at its poles.
4. Expand $f(z) = tanz$ in Taylor's series about $z = 0$.
5. Expand $f(z) = \frac{z+3}{z(z^2-z-2)}$ for $|z| > 2$.
6. Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x^2$.
7. Define following (i) Singular point (ii) isolated Singular point (iii) Essential Singularity

   (iv) Removable Singularity (v) pole of analytic function

8. Evaluate $\int_C \frac{z^3-sin3z}{\left(z-\frac{\pi}{2}\right)^3}dz$, where C is the circle $|z| = 2$ using Cauchy's integral formula.

9. Evaluate $\int_C \frac{sin\pi z^2+cos\pi z^2}{(z-1)(z-2)}dz$, where C is the circle $|z| = 3$ using Cauchy's integral formula.
10. Evaluate $\int_{1-i}^{2+i}(2x + 1 + iy)dz$ along the path $(1 - i)\ to\ (2 + i)$.

11. Obtain the expansion for $\sin\left[\frac{1}{z-1}\right]$ which is valid in $1 < |z| < \infty$
12. Evaluate $\int_C \frac{(2z+1)^2}{z^8(4z^3+z)}\, dz$ over a unit circle C.
13. Evaluate $\int_C (x-2y)dx + (y^2 - x^2)dy$ where C is the boundary of the first quadrant of the circle $x^2 + y^2 = 4$
14. Using Cauchy integral formula, find $\int_C \frac{e^{2z}}{(z+1)^3}$ dz , where C is the curve $|z|$ = 2.
15. Evaluate $\int_C (x^2 - iy^2)\ dz$ along a straight line from (0,0) to (0,1) and then from (0,1) to (2,1).
16. Find Laurent's series of $\frac{z}{(z-1)(z-2)}$ about: a) $|z| < 1$ b) $|z| > 1$ c) $1 < |z| < 2$
17. Evaluate $\int_C \frac{1}{z^8(z+4)}$ dz, where C is the circle $|z|$ = 2
18. Evaluate $\int_C \frac{z+4}{z^2+2z+5}$ dz where C is the circle (i) $|z+1-i|$ = 2 (i) $|z+1+i|$ = 2
19. If C is the boundary of the square with vertices at the points z=0, z=1, z=1+i, z=I show that $\int_C (3z+1)dz = o.$
20. Represent the function f(z) = $\frac{1}{z(z+2)^3(z+1)^2}$ in Laurent series within $\frac{5}{4} \le |z| \le \frac{7}{4}$
21. Evaluate $\int_C \frac{dz}{z \sin z}$ where C is the unit circle with centre at the origin.

(13) *LIST OF TOPICS FOR STUDENT'S SEMINARS*

1. To find backward and forward differences
2. To find Integrations by using Numerical Methods.
3. Complex variables and C-R eqns.
4. Mobius Transformations.

(14) *STEP/COURSE MATERIAL*

(15) *EXPERT LECTURES WITH TOPICS & SCHEDULES*

1. Guest lecture on "Laplace Transforms, Numerical Methods and Complex Variables" tentatively scheduled in between 26/04/2021 and 30/04/2021.

□□□□***THE END*** □□□